Lecture Notes

for

hybrid Parameters

(PHYS4008: Electronics)

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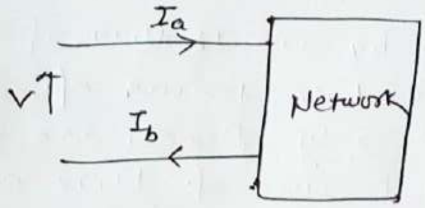
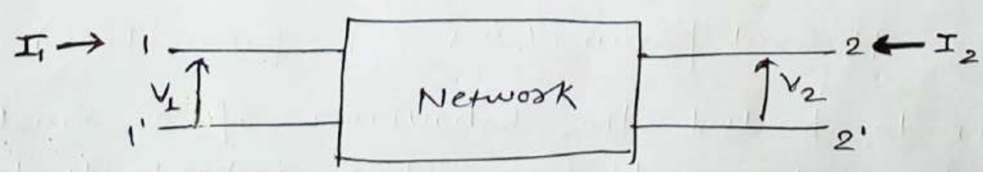
TWO PORT NETWORK & Hybrid parameters

* A pair of terminals at which a signal may enter or leave a network is called a port.
* A network having only one such pair of terminals is termed as one-port network or two-terminal network.
* 

A one-port Network

$\boxed{I_a = I_b} = I$

* We may define input impedance as $\boxed{Z_{in} = \frac{V}{I}}$
* or input admittance $\boxed{Y_{in} = \frac{I}{V}}$
* A one port network may contain either only the passive elements or the combination of passive elements with the dependent source.
* A network with four terminals is referred as two-port network or four terminal network.



The terminals 1 and 1' represent the input port and terminals 2 and 2' the output network. In some network, 1' and 2' may be common. Measurement can be made only at the input and output ports but not between 1 and 2 or betn terminals 1' and 2'.

→ The transformer and amplifiers are modelled in terms of two-port network parameters.

→ A two port network can be fully described by four variables which are the port voltages and currents, namely, V_1, V_2, I_1, I_2. Any pair may be arbitrarily chosen as the independent variables, and other two as dependent variables.

⇒ There are six combinations by which two of the four variables can be expressed in terms of the remaining two variables. Only three are generally used for circuit analysis because of their ease of measurement.

i) Z parameters

$$\boxed{\begin{aligned} V_1 &= Z_{11}(s) I_1 + Z_{12}(s) I_2 \\ V_2 &= Z_{21}(s) I_1 + Z_{22}(s) I_2 \end{aligned}}$$

ii) y parameters

$$\boxed{\begin{aligned} I_1 &= y_{11} V_1 + y_{12} V_2 \\ I_2 &= y_{21} V_1 + y_{22} V_2 \end{aligned}}$$

iii) h parameters

$$\boxed{\begin{aligned} V_1 &= h_{11} I_1 + h_{12} V_2 \\ I_2 &= h_{21} I_1 + h_{22} V_2. \end{aligned}}$$

Hybrid parameters (h parameters)

In order to predict the behaviour of a small-signal transistor amplifier, it is important to know its operating characteristics e.g. input impedance, output impedance, voltage gain etc. In our discussion so far, these characteristics were determined by using current gain ($\alpha, \beta, \ldots$) and circuit resistances. This method has two advantages

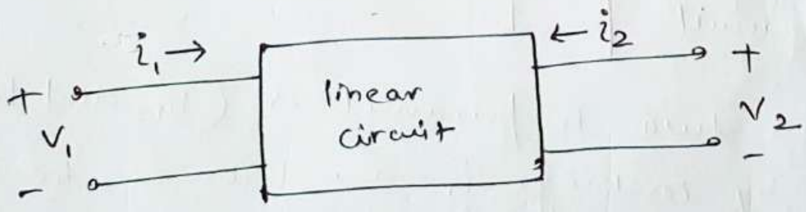
i) the values of circuit components are readily available.

ii) the procedure followed is easily understood.

→ The major drawback of the above method is that accurate results cannot be obtained. It is because the input and output circuits of a transistor amplifier are not completely independent. e.g. output current is affected by the value of load resistance rather than being constant at the values βI_b. Similarly, output voltage has an effect on the input circuit so that changes in the output causes changes in the input. One of the methods that takes into account all the effect in transistor amplifier is h-parameters approach.

* Every linear circuit having input and output terminals can be analysed by four parameters (one measured in ohm, one in mho and two dimensionless) called hybrid or h-parameters.

$$v_1 = h_{11} i_1 + h_{12} v_2 \longrightarrow \text{(i)}$$
$$i_2 = h_{21} i_1 + h_{22} v_2 \longrightarrow \text{(ii)}$$



e.g. $v_1 = 10 i_1 + 6 v_2$

$i_2 = 4 i_1 + 3 v_2$

$h_{11} = 10 \Omega$

$h_{12} = 6$

$h_{21} = 4$

$h_{22} = 3$ mho

* A linear circuit is one in which resistance, inductance and capacitance remain fixed when voltage across them changes

Determination of h parameters.

i) If we short-circuit the output terminals, i.e. $\boxed{V_2 = 0}$

The eqn's (i) and (ii) becomes

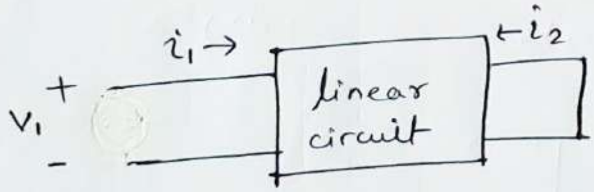
$$V_1 = h_{11} i_1 + h_{12} \times 0$$
$$i_2 = h_{21} i_1 + h_{22} \times 0$$

$$\therefore \boxed{h_{11} = \frac{V_1}{i_1}} \quad (\text{for } V_2 = 0 \text{ i.e. output shorted})$$

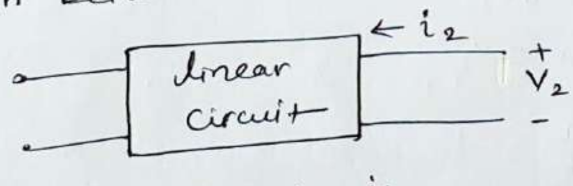
$$\therefore \boxed{h_{21} = \frac{i_2}{i_1}} \quad (\text{for } V_2 = 0 \text{ i.e. output shorted})$$

Since h_{11} is a ratio of voltage and current (i.e. $\frac{V_2}{i_1}$) it is an impedance and is called input impedance with output shorted.

Similarly, h_{21} is ratio of output and input current (i.e. $\frac{i_2}{i_1}$) it will be dimensionless and is called current gain with output shorted.



ii) The other two h parameters (h_{12} and h_{22}) can be found by making $i_1 = 0$. This can be done as shown below.

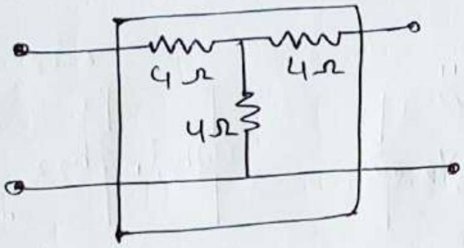


$$V_1 = h_{11} \times 0 + h_{12} V_2$$
$$i_2 = h_{21} \times 0 + h_{22} V_2$$

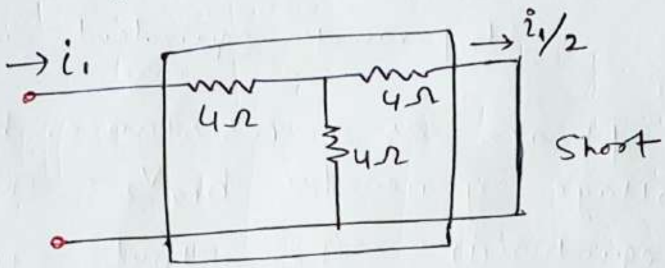
$$\therefore \boxed{h_{12} = \frac{V_1}{V_2}} \quad \left(\begin{array}{c}\text{input open} \\ \text{i.e. } i_1 = 0\end{array}\right) \leftarrow \text{Reverse transfer voltage gain with open input terminals}$$

$$\therefore \boxed{h_{22} = \frac{i_2}{V_2}} \quad \left(\begin{array}{c}\text{input open} \\ \text{i.e. } i_1 = 0\end{array}\right) \leftarrow \text{Output admittance with input terminals open}$$

Q Find the h parameters of the following circuit.



Solⁿ



input impedance, $h_{11} = 4 + 4 \| 4$

$$h_{11} = 4 + \frac{4 \times 4}{4+4} = 6\,\Omega$$

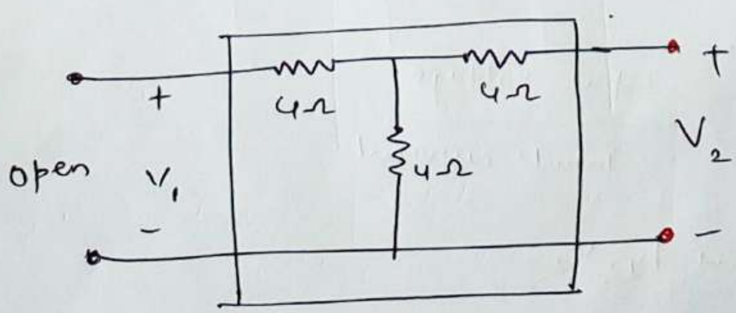
$$\therefore \boxed{h_{11} = 6\,\Omega}$$

in the above circuit

$$i_2 = -\frac{i_1}{2}$$

$$i_2 = -0.5\, i_1$$

$$\therefore \boxed{h_{21} = \frac{i_2}{i_1} = -0.5}$$



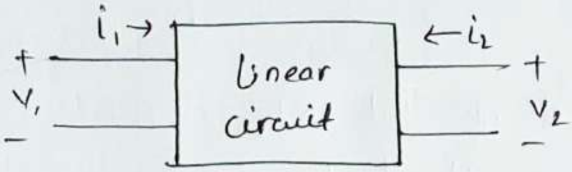
$$V_1 = \frac{V_2}{2} = 0.5\, V_2$$

$$\boxed{h_{12} = \frac{V_1}{V_2} = 0.5}$$

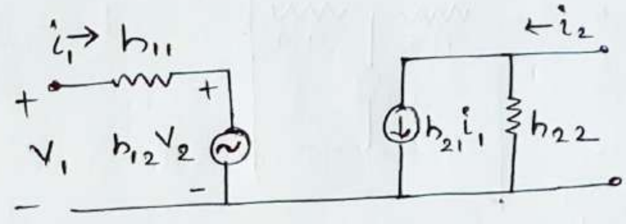
Now the output impedance looking into output terminals with open input terminals is 8 Ω

$$\therefore \boxed{h_{22} = \frac{1}{8} = 0.125 \text{ mho}}$$

h parameter equivalent circuit.



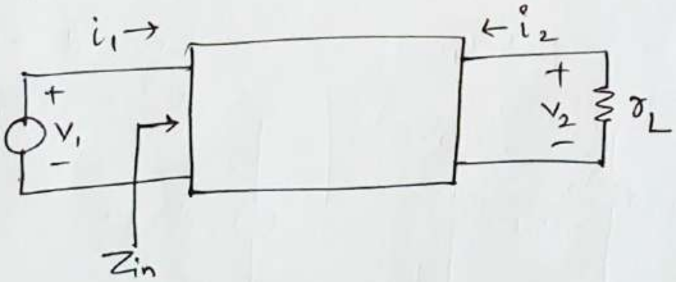
Linear circuit



h parameter equivalent circuit

The input circuit appears as a resistance h_{11} in series with a voltage generator $h_{12}V_2$. This circuit is derived from equation (i). The output circuit involve two components; a current generator $h_{21}i_1$ and shunt resistance h_{22} and is derived from eqn (ii).

i) input impedance: Consider a linear circuit with a load resistance r_L across its terminals as shown below.



$$Z_{in} = \frac{V_1}{i_1} = \frac{\text{input voltage}}{\text{input current}}$$

$$\because V_1 = h_{11} i_1 + h_{12} V_2$$

$$\therefore \quad Z_{in} = \frac{h_{11} i_1 + h_{12} V_2}{i_1} = \boxed{h_{11} + \frac{h_{12} V_2}{i_1}} \rightarrow (i)$$

$$\because i_2 = \frac{-V_2}{r_L} \quad \text{and} \quad i_2 = h_{21} i_1 + h_{22} V_2$$

$$\therefore \quad \frac{-V_2}{r_L} = h_{21} i_1 + h_{22} V_2$$

$$\therefore \quad -h_{21} i_1 = h_{22} V_2 + \frac{V_2}{r_L} = V_2\left(h_{22} + \frac{1}{r_L}\right)$$

$$\therefore \quad \boxed{\frac{V_2}{i_1} = \frac{-h_{21}}{h_{22} + \frac{1}{r_L}}} \longrightarrow \text{(ii)}$$

from eq^ns (i) and (ii)

$$\boxed{Z_{in} = h_{11} - \frac{h_{12} h_{21}}{h_{22} + \frac{1}{r_L}}}$$

If either of h_{12} or r_L is very small, the second term can be neglected.

$$\therefore \quad \boxed{Z_{in} \simeq h_{11}}$$

ii) Current gain: $A_i = \frac{i_2}{i_1}$

$$\because \quad i_2 = h_{21} i_1 + h_{22} V_2$$

$$\text{and} \quad V_2 = -i_2 r_L$$

$$\therefore \quad i_2 = h_{21} i_1 - h_{22} i_2 r_L$$

$$\therefore \quad \frac{i_2}{i_1} = \frac{h_{21}}{1 + h_{22} r_L}$$

$$\therefore \quad \boxed{A_i = \frac{h_{21}}{1 + h_{22} r_L}}$$

If $h_{22} r_L << 1$, $A_i \simeq h_{21}$ (i.e. $r_L <<$ output resistance $\left(\frac{1}{h_{22}}\right)$)

Under such condition, most of generator current bypass the circuit output resistance in favour of r_L.

iii) Voltage gain :

$$A_v = \frac{v_2}{v_1}$$

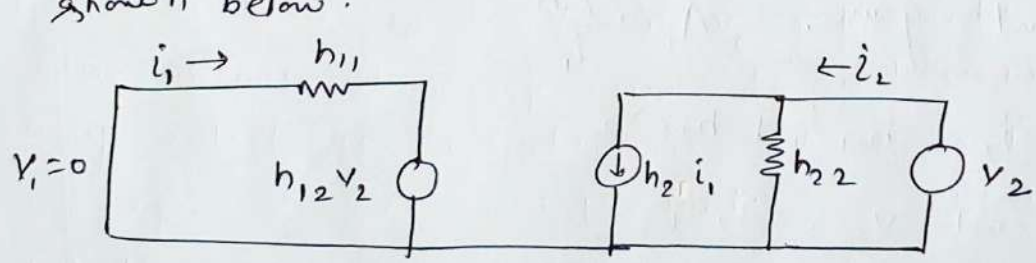
$$= \frac{v_2}{i_1 z_{in}} \quad (\because v_1 = i_1 z_{in})$$

While developing expression for input resistance, we found that

$$\frac{v_2}{i_1} = \frac{-h_{21}}{h_{22} + \frac{1}{r_L}}$$

$$\therefore \boxed{A_v = \frac{-h_{21}}{Z_{in}\left(h_{22} + \frac{1}{r_L}\right)}}$$

iv) Output impedance : In order to find the output impedance, remove the load r_L, set the voltage $v_1 = 0$ and connect a generator of v_2 at output terminals. The h parameters equivalent circuit is shown below.



$$Z_{out} = \frac{v_2}{i_2}$$

$$0 = h_{11} i_1 + h_{12} v_2$$

$$\therefore \boxed{i_1 = \frac{-h_{12} v_2}{h_{11}}}$$

$$\because i_2 = h_{21} i_1 + h_{22} v_2$$

$$i_2 = h_{21}\left(\frac{-h_{12} v_2}{h_{11}}\right) + h_{22} v_2$$

$$i_2 = \frac{-h_{21} h_{12} v_2}{h_{11}} + h_{22} v_2$$

Dividing throughout by V_2, we have

$$\frac{i_2}{V_2} = \frac{-h_{21}h_{12}}{h_{11}} + h_{22}$$

$$Z_{out} = \frac{V_2}{i_1} = \boxed{\frac{1}{h_{22} - \frac{h_{21}h_{12}}{h_{11}}}}$$

⇒ One of the first concerns in the sinusoidal ac analysis of transistor networks is the magnitude of the input signal. Study of small signal operation may be done either graphically or by using small signal equivalent circuit for the BJT operating in active region. However second method is more convenient.

⇒ The large signal operation may be best studied graphically because of involvement of certain non-linear operation on it.

⇒ In transistor amplifier analysis, z and Y parameters were used earlier. But now hybrid or the h-parameter alone are used in a transistor circuit analysis and therefore, only the h-parameters will be taken here for discussion.

⇒ Notations : The convenient alternative subscript notations recommended by the IEEE standards are given below.

$i = 11 =$ input

$o = 22 =$ output

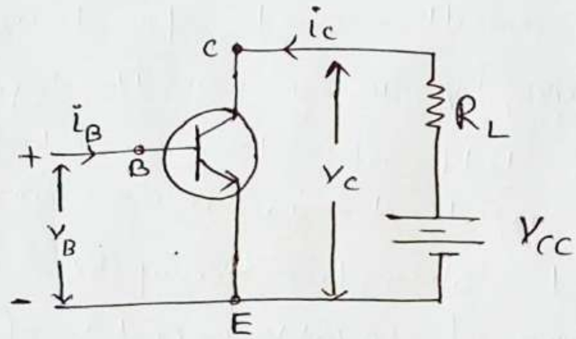
$r = 12 =$ reverse transfer

$f = 21 =$ forward transfer

In case of transistors, another subscript (b, e, or c) is added to designate the type of transistor configuration. for example: $h_{ie} = h_{11e} =$ Input resistance in common emitter configuration.

Many transistor models have been proposed. each one having its particular merit and demerits. The transistor model in terms of h-parameters, which are real number at audio frequency, are easy to measure; can also be obtained from static characteristics of a transistor. furthermore, a set of h-parameters is specified for many transistor by the manufacturers.

To derive a hybrid model for a transistor, let us consider the basic CE amplifier circuit as shown below.



The variables i_B, i_C, V_B and V_C represent the total instantaneous values of currents and voltages. We may select the input current i_B and output voltage V_C as independent variables. Since input voltage V_B is some function f_1 of i_B and V_C and output current i_c is another function f_2 of i_B and V_c, we may write

$$V_B = f_1(i_B, V_c) \longrightarrow (i)$$

$$i_c = f_2(i_B, V_c) \longrightarrow (ii)$$

Making a Taylor's series expansion of eqns (i) and (ii) about zero signal operating point (I_B, V_C) and neglecting higher order terms we have

$$\Delta v_B = \left.\frac{\delta f_1}{\delta i_B}\right|_{V_C} \Delta i_B + \left.\frac{\delta f_1}{\delta v_C}\right|_{I_B} \Delta v_C \longrightarrow \text{(iii)}$$

$$\Delta i_C = \left.\frac{\delta f_2}{\delta i_B}\right|_{V_C} \Delta i_B + \left.\frac{\delta f_2}{\delta v_C}\right|_{I_B} \Delta v_C \longrightarrow \text{(iv)}$$

The quantities Δv_B, Δv_C, Δi_B and Δi_C represent the small signal (incremental) base and collector voltages and currents and may be represented as v_b, v_c, i_b and i_c respectively as per standard notations. We may now write eqns (iii) and (iv) as below.

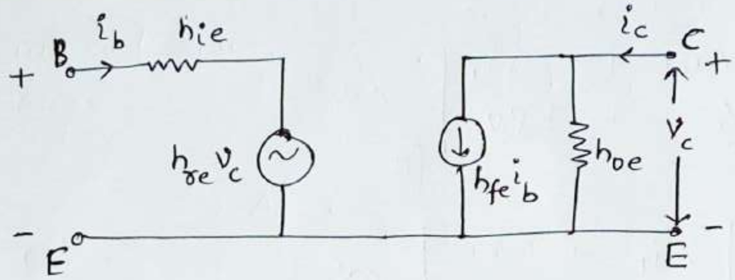
$$\boxed{\begin{aligned} v_b &= h_{ie} i_b + h_{re} v_c \\ i_c &= h_{fe} i_b + h_{oe} v_c \end{aligned}} \quad \begin{aligned} &\longrightarrow \text{(v)} \\ &\longrightarrow \text{(vi)} \end{aligned}$$

where $h_{ie} = \left.\frac{\delta f_1}{\delta i_B}\right|_{V_C} = \left.\frac{\delta v_B}{\delta i_B}\right|_{V_C} = \left.\frac{v_b}{i_b}\right|_{V_C = 0}$

$$h_{fe} = \left.\frac{\delta f_2}{\delta i_B}\right|_{V_C} = \left.\frac{\delta i_C}{\delta i_B}\right|_{V_C} = \left.\frac{i_c}{i_b}\right|_{V_C = 0}$$

$$h_{re} = \left.\frac{\delta f_1}{\delta v_C}\right|_{I_B} = \left.\frac{\delta v_B}{\delta v_C}\right|_{I_B} = \left.\frac{v_b}{v_c}\right|_{I_B = 0}$$

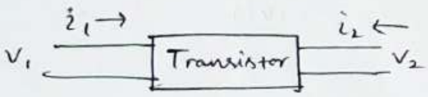
$$h_{oe} = \left.\frac{\delta f_2}{\delta v_C}\right|_{I_B} = \left.\frac{\delta i_C}{\delta v_C}\right|_{I_B} = \left.\frac{i_c}{v_c}\right|_{I_B = 0}$$

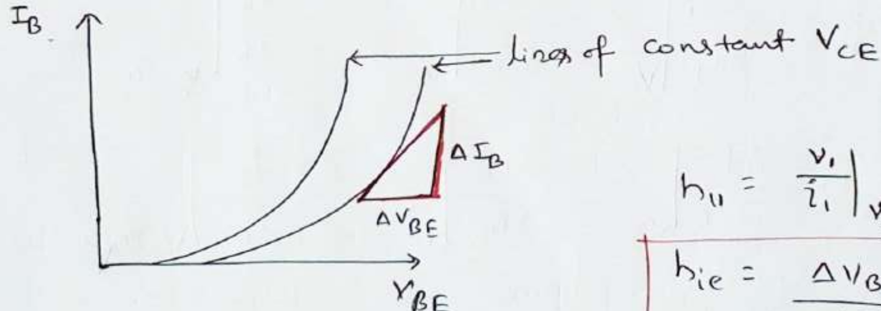


$$\begin{vmatrix} v_b \\ i_c \end{vmatrix} = \begin{vmatrix} h_{ie} & h_{re} \\ h_{fe} & h_{oe} \end{vmatrix} \begin{vmatrix} i_b \\ v_c \end{vmatrix}$$

Graphical determination of h-parameters

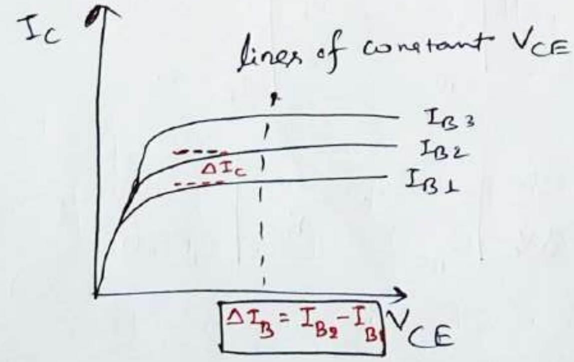
The characteristics of a transistor in CE mode is given below for determining the h-parameters.





$$h_{11} = \left.\frac{v_1}{i_1}\right|_{v_2 = 0}$$

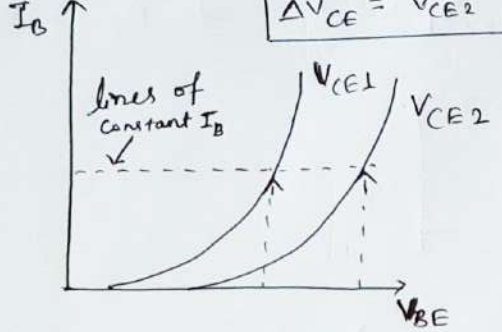
$$\boxed{h_{ie} = \left.\frac{\Delta V_{BE}}{\Delta I_B}\right|_{V_{CE} = \text{constant}}}$$



$$h_{21} = \left.\frac{i_2}{i_1}\right|_{v_2 = 0}$$

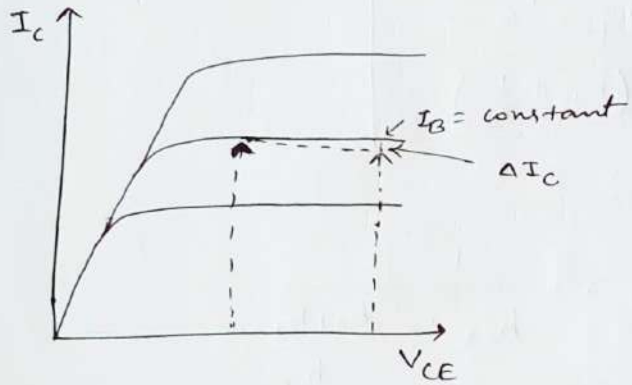
$$\boxed{h_{fe} = \left.\frac{\Delta I_C}{\Delta I_B}\right|_{V_{CE} = \text{constant}}}$$

$$\boxed{\Delta V_{CE} = V_{CE2} - V_{CE1}}$$



$$h_{12} = \left.\frac{v_1}{v_2}\right|_{i_1 = 0}$$

$$\boxed{h_{re} = \left.\frac{\Delta V_{BE}}{\Delta V_{CE}}\right|_{I_B = \text{Constant}}}$$



$$h_{22} = \left.\frac{i_2}{v_2}\right|_{i_1 = 0}$$

$$\boxed{h_{oe} = \left.\frac{\Delta I_C}{\Delta V_{CE}}\right|_{I_B = \text{constant}}}$$

Transistor circuit performance in h parameters

As discussed earlier, the expression for input impedance voltage gain etc. in terms of h parameters for general circuit analysis apply equally for the transistor analysis. However, it is convenient to rewrite them in standard transistor h parameter nomenclature.

i) Input Impedance: $Z_{in} = h_{11} - \dfrac{h_{12} h_{21}}{h_{22} + \frac{1}{r_L}}$

$$\therefore \quad \boxed{Z_{in} = h_{ie} - \frac{h_{re} h_{fe}}{h_{oe} + \frac{1}{r_L}}}$$

r_L = ac load seen by the transistor

(ii) current gain: $A_i = \dfrac{h_{21}}{1 + h_{22} r_L}$

$$\boxed{A_i = \frac{h_{fe}}{1 + h_{oe} r_L}}$$

(iii) Voltage gain:

$$A_v = \frac{-h_{21}}{Z_{in}\left(h_{22} + \frac{1}{r_L}\right)}$$

$$\boxed{A_v = \frac{-h_{fe}}{Z_{in}\left(h_{oe} + \frac{1}{r_L}\right)}}$$

(iv) output Impedance;

$$Z_{out} = \frac{1}{h_{22} - \dfrac{h_{21} h_{12}}{h_{11}}}$$

$$\boxed{Z_{out} = \frac{1}{h_{oe} - \dfrac{h_{fe} h_{re}}{h_{ie}}}}$$

If the transistor is connected in a circuit to form a single stage amplifier; then output impedance of the stage $= Z_{out} \| r_L$ where $r_L = R_C \| R_L$

Q1 A transistor used in CE arrangement has the following set of h parameters when the d.c. operating point is $V_{CE} = 10$ V and $I_C = 1$ mA

$h_{ie} = 2000\,\Omega$, $h_{oe} = 10^{-4}$ mho, $h_{re} = 10^{-3}$, $h_{fe} = 50$

Determine the (i) input resistance (ii) current gain (iii) voltage gain. The a.c load seen by the transistor is $r_L = 600\,\Omega$. What will be approximate values.

Soln

i) $Z_{in} = h_{ie} - \dfrac{h_{re} h_{fe}}{h_{oe} + \frac{1}{r_L}} = 2000 - \dfrac{10^{-3} \times 50}{10^{-4} + \frac{1}{600}}$

$= 2000 - 28$

$= 1972\ \Omega$

$\therefore\ Z_{in} \simeq h_{ie} = 2000\ \Omega$

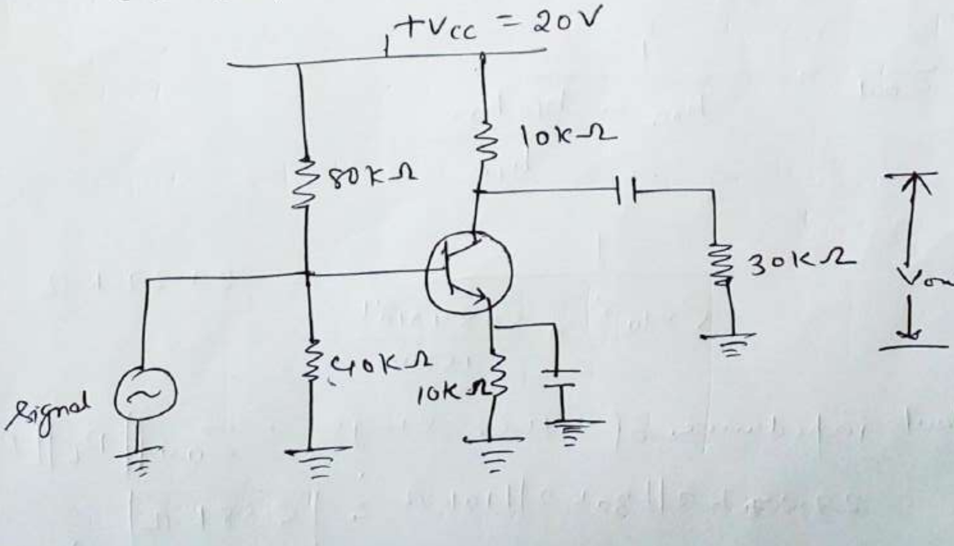
ii) $A_i = \dfrac{h_{fe}}{1 + h_{oe} r_L} = \dfrac{50}{1 + 600 \times 10^{-4}} = 47$

If $h_{oe} r_L << 1$ then $A_i \simeq h_{fe} = 50$

(iii) $A_v = \dfrac{-h_{fe}}{Z_{in}\left(h_{oe} + \frac{1}{r_L}\right)} = \dfrac{-50}{1972\left(10^{-4} + \frac{1}{600}\right)} = -14.4$

-ve sign indicates that there is 180° phase shift between input and output.

Q2. Figure shows the transistor amplifier in CE arrangement. The h parameters are as under
$h_{ie} = 1500\ \Omega$, $h_{fe} = 50$, $h_{re} = 4 \times 10^{-4}$, $h_{oe} = 5 \times 10^{-5}\ \mho$
Find the (i) a.c input impedance (ii) voltage gain and (iii) output impedance



Soln: The a.c. load r_L seen by the transistor is equivalent of the parallel combination of R_C (= 10 kΩ) and R_L (= 30 kΩ) i.e.

$$r_L = \frac{R_C R_L}{R_C + R_L} = \frac{10 \times 30}{10 + 30} = 7.5\,k\Omega$$

i) $$Z_{in} = h_{ie} - \frac{h_{re} h_{fe}}{h_{oe} + \frac{1}{r_L}}$$

$$= 1500 - \frac{4 \times 10^{-4} \times 50}{5 \times 10^{-5} + \frac{1}{7500}} = 1390\,\Omega$$

Input impedance of the stage

$$= 80 \times 10^3 \,||\, 40 \times 10^3 \,||\, 1390$$

$$= \boxed{1320\,\Omega}$$

ii) $$A_v = \frac{-h_{fe}}{Z_{in}\left(h_{oe} + \frac{1}{r_L}\right)} = \frac{-50}{1390\left(5 \times 10^{-5} + \frac{1}{7500}\right)}$$

$$\boxed{A_v = -196}$$

iii) $$Z_{out} = \frac{1}{h_{oe} - \frac{h_{fe} h_{re}}{h_{ie}}}$$

$$= \frac{1}{5 \times 10^{-5} - \frac{50 \times 4 \times 10^{-4}}{1500}} = 27.27\,k\Omega$$

output impedance of the stage $= Z_{out} \,||\, R_L \,||\, R_C$

$$= 27.27\,k\Omega \,||\, 30\,k\Omega \,||\, 10\,k\Omega = \boxed{5.88\,k\Omega}$$

Ans

References:

1. Op-Amps and Linear Integrated Circuits by R. A. Gayakwad
2. Linear Integrated Circuits by D. R. Choudhury and S. B. Jain
3. Electronics Fundamentals and Applications by D. Chattopadhyay and P.C. Rakshit
4. Electronic Devices and Circuits by J. Millman and C.C. Halkias
5. Integrated Electronics by J. Millman and C.C. Halkias