Lecture-II Programme: M. Sc. Physics Sem-II Paper code: PHYS4007

Title: ELECTRODYNAMICS

Wave guide: Rectangular Wave Guide [TEM] Mode

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OBJECTIVES

The main objective of the lecture is to present the basic concepts of waveguide that are very significant from the their applications point of view of in telecommunication.

Introduction

A transmission line is used to guide EM energy from one point (generator) to another (load) whereas a waveguide is one more way of getting the similar purpose. But, a waveguide differs from a transmission line in few respects. In the first way, a transmission line can carry only a transverse electromagnetic (TEM) wave, while a waveguide can maintain several probable field configurations.

Second, in a microwave frequencies range (3–300 GHz), transmission lines become incompetent as a consequence of skin effect and dielectric losses; waveguides are utilized at that range of frequencies to achieve more bandwidth and lesser signal attenuation. In addition, a transmission line may function from dc (f = 0) to a very high frequency.

A waveguide that can work only above a certain frequency is known as a the cut off frequency and so it works as a high-pass filter. hence, we can say that waveguides cannot transmit dc, and they become extremely high at frequencies lower microwave frequencies. Even though a waveguide may suppose any random but regular cross section, general waveguides are either rectangular or circular. Some regular shape Waveguides are shown in Figure 1.

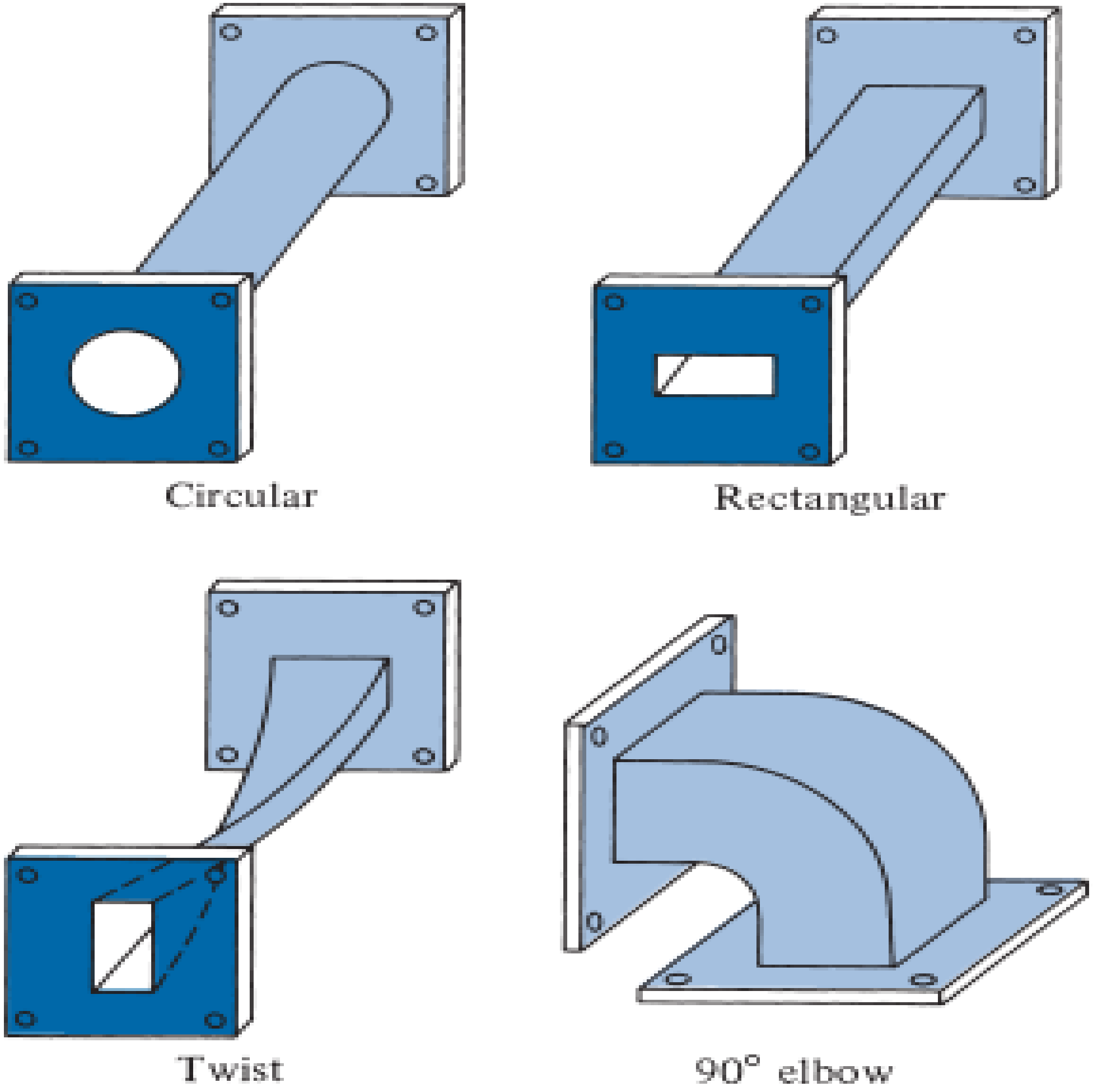

Figure 1: Some regular shape waveguides

Here, we will discuss only hollow rectangular waveguides. By considering lossless waveguides, we shall utilize Maxwell's equations with the suitable boundary conditions to get different modes of wave propagation and the analogous E and H fields. When we close both ends of a waveguide, a cavity is created. We will also assume optical fiber guide, which is basic to optical data communications.

RECTANGULAR WAVEGUIDES

- **Here we are starting the analysis for a rectangular waveguide. For this , we suppose a rectangular waveguide shown in Figure 2, of geometrical inner dimensions are *a* and *b*. We also shall assume that the waveguide is filled with a source free (ρ_v = 0, J = 0) lossless dielectric material ($\sigma \approx 0$) and its dimensions are made by a perfectly conducting ($\sigma_c \approx \infty$) material . From our previous information we recall that for a lossless medium, the form of Maxwell's equations in phasor form become**

$$\nabla^2 \mathbf{E}_s + k^2 \mathbf{E}_s = 0 \qquad \text{-------------- [1]}$$

$$\nabla^2 \mathbf{H}_s + k^2 \mathbf{H}_s = 0 \qquad \text{-------------- [2]}$$

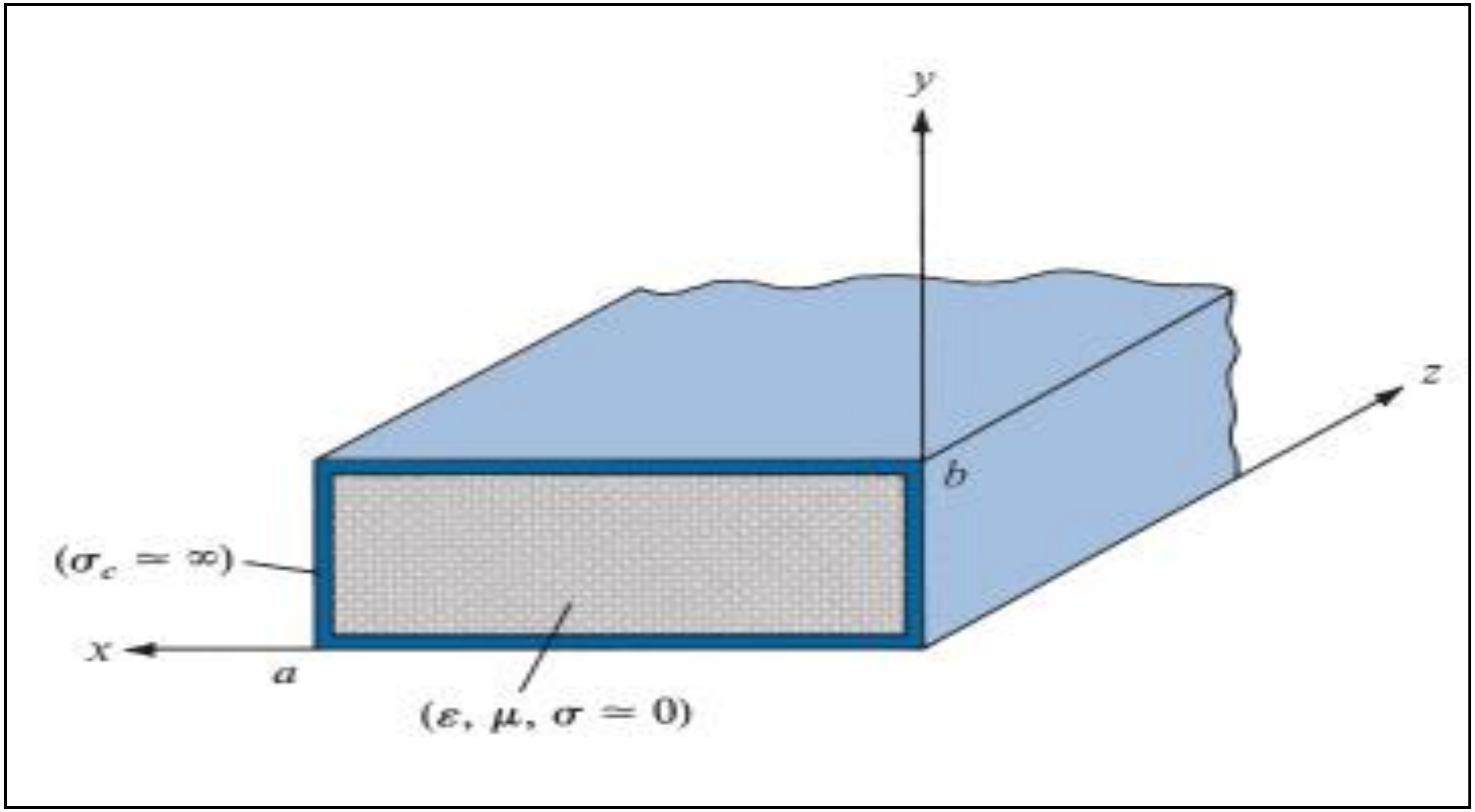

Figure 2: A rectangular waveguide with perfectly conducting walls, filled with a lossless material.

- **Where k is defined by the following equation,**

$$k = \omega\sqrt{\mu\varepsilon} \quad \text{-------------- [3]}$$

- **If time factor is assumed and the forms of field vectors say**

$$\mathbf{E}_s = (E_{xs}, E_{ys}, E_{zs}) \quad \text{and} \quad \mathbf{H}_s = (H_{xs}, H_{ys}, H_{zs})$$

both of eqs. (1) and (2) involves three scalar Helmholtz equations. In other words, to find forms of the E and H fields, we have to solve six scalar equations. For the *z*-component, for example, eq. (1) becomes

$$\frac{\partial^2 E_{zs}}{\partial x^2} + \frac{\partial^2 E_{zs}}{\partial y^2} + \frac{\partial^2 E_{zs}}{\partial z^2} + k^2 E_{zs} = 0 \qquad \text{---------- [4]}$$

which is a partial differential equation. The solutions of this equation can be solved by separation of variables (product solution). So we let

$$E_{zs}(x, y, z) = X(x)\, Y(y)\, Z(z) \qquad \text{---------- [5]}$$

where *X*(*x*), *Y*(*y*), and *Z*(*z*) are functions of *x*, *y*, and *z*, respectively. Substituting eq. (5) into eq. (4) and dividing by *XYZ* gives

$$\frac{X''}{X} + \frac{Y''}{Y} + \frac{Z''}{Z} = -k^2 \qquad \text{---------- [6]}$$

Since the variables are independent, each term in eq. (6) must be constant, so the equation can be written as

$$-k_x^2 - k_y^2 + \gamma^2 = -k^2 \quad \text{---------} \quad [7]$$

where *kx* , *ky*, and γ are constants. Thus, eq. (6) is separated as

$$X'' + k_x^2 X = 0 \quad \text{---------} \quad [8]$$

$$Y'' + k_y^2 Y = 0 \quad \text{---------} \quad [9]$$

$$Z'' - \gamma^2 Z = 0 \quad \text{-----------} \quad [10]$$

The selection of γ^2 is due to the realization that the guided waves propagate along the axis *z* in the +ve or -ve direction, and the propagation may result in E_{zs} and H_{zs} that move toward zero as $z \pm \infty$`. The following solutions of these differential equations can be written as-

$$X(x) = c_1 \cos k_x x + c_2 \sin k_x x \quad \text{-----------} \quad [11]$$

$$Y(y) = c_3 \cos k_y y + c_4 \sin k_y y \quad \text{-----------} \quad [12]$$

$$Z(z) = c_5 e^{\gamma z} + c_6 e^{-\gamma z} \quad \text{-----------} \quad [13]$$

$$E_{zs}(x,y,z) = (c_1 \cos k_x x + c_2 \sin k_x x)(c_3 \cos k_y y + c_4 \sin k_y y)\,(c_5 e^{\gamma z} + c_6 e^{-\gamma z}) \qquad [14]$$

if we suppose that the wave propagates along the waveguide in the *z*-direction, the multiplicative constant $c_5 = 0$ because the wave has to be finite at infinity [$E_{zs}(x, y, z = \infty) = 0$]. Hence eq. (14) takes the form-

$$E_{zs}(x,y,z) = (A_1 \cos k_x x + A_2 \sin k_x x)(A_3 \cos k_y y + A_4 \sin k_y y)e^{-\gamma z} \qquad [15]$$

where $A_1 = c_1 c_6$, $A_2 = c_2 c_6$, $A_3 = c_3 c_6$, and $A_4 = c_4 c_6$.

Now by using same procedure, we get the solution of the *z*-component of for the magnetic field vector from eq. (2) as

$$H_{zs}(x,y,z) = (B_1 \cos k_x x + B_2 \sin k_x x)(B_3 \cos k_y y + B_4 \sin k_y y)e^{-\gamma z} \qquad [16]$$

in its place of solving for other field components such as *E_{xs}*, *E_{ys}*, *H_{xs}*, and *H_{ys}* in eqs. (1) and (2) in the same manner, it is more convenient to use Maxwell's equations to determine them from *Ezs* and *Hzs*. From

$$\nabla \times \mathbf{E}_s = -j\omega\mu\mathbf{H}_s \qquad \textbf{and} \qquad \nabla \times \mathbf{H}_s = j\omega\varepsilon\mathbf{E}_s$$

We obtained -

$$\frac{\partial E_{zs}}{\partial y} - \frac{\partial E_{ys}}{\partial z} = -j\omega\mu H_{xs} \qquad \text{------------ [17]}$$

$$\frac{\partial H_{zs}}{\partial y} - \frac{\partial H_{ys}}{\partial z} = j\omega\varepsilon E_{xs} \qquad \text{------------ [18]}$$

$$\frac{\partial E_{xs}}{\partial z} - \frac{\partial E_{zs}}{\partial x} = -j\omega\mu H_{ys} \qquad \text{------------ [19]}$$

$$\frac{\partial H_{xs}}{\partial z} - \frac{\partial H_{zs}}{\partial x} = j\omega\varepsilon E_{ys} \qquad \text{------------ [20]}$$

$$\frac{\partial E_{ys}}{\partial x} - \frac{\partial E_{xs}}{\partial y} = -j\omega\mu H_{zs} \qquad \text{------------ [21]}$$

$$\frac{\partial H_{ys}}{\partial x} - \frac{\partial H_{xs}}{\partial y} = j\omega\varepsilon E_{zs} \qquad \text{------------ [22]}$$

- **We will now express E_{xs}, E_{ys}, H_{xs}, and H_{ys} in terms of E_{zs} and H_{zs}. For E_{xs}, for example, we combine eqs. (18) and (19) and obtain**

$$j\omega\varepsilon E_{xs} = \frac{\partial H_{zs}}{\partial y} + \frac{1}{j\omega\mu}\left(\frac{\partial^2 E_{xs}}{\partial z^2} - \frac{\partial^2 E_{zs}}{\partial x \partial z}\right) \quad \text{-----------} \quad [23]$$

From equations (15) and (16), it is obvious that all field components vary with z according to $e^{-\gamma z}$, that is,

$$E_{zs} \sim e^{-\gamma z}, \quad E_{xs} \sim e^{-\gamma z}$$

$$\frac{\partial E_{zs}}{\partial z} = -\gamma E_{zs}, \quad \frac{\partial^2 E_{xs}}{\partial z^2} = \gamma^2 E_{xs}$$

$$j\omega\varepsilon E_{xs} = \frac{\partial H_{zs}}{\partial y} + \frac{1}{j\omega\mu}\left(\gamma^2 E_{xs} + \gamma \frac{\partial E_{zs}}{\partial x}\right)$$

$$-\frac{1}{j\omega\mu}\left(\gamma^2 + \omega^2\mu\varepsilon\right) E_{xs} = \frac{\gamma}{j\omega\mu}\frac{\partial E_{zs}}{\partial x} + \frac{\partial H_{zs}}{\partial y}$$

Thus, if we let $h^2 = \gamma^2 + \omega^2\mu\varepsilon = \gamma^2 + k^2$,

$$E_{xs} = -\frac{\gamma}{h^2}\frac{\partial E_{zs}}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_{zs}}{\partial y}$$

Similar treatments of eqs. (17-22) provide the forms for E_{ys}, H_{xs}, and H_{ys} in terms of E_{zs} and H_{zs}. And these are as follows-

$$E_{xs} = -\frac{\gamma}{h^2}\frac{\partial E_{zs}}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_{zs}}{\partial y} \quad \text{----------} \quad [24]$$

$$E_{ys} = -\frac{\gamma}{h^2}\frac{\partial E_{zs}}{\partial y} + \frac{j\omega\mu}{h^2}\frac{\partial H_{zs}}{\partial x} \quad \text{----------} \quad [25]$$

$$H_{xs} = \frac{j\omega\varepsilon}{h^2}\frac{\partial E_{zs}}{\partial y} - \frac{\gamma}{h^2}\frac{\partial H_{zs}}{\partial x} \quad \text{----------} \quad [26]$$

$$H_{ys} = -\frac{j\omega\varepsilon}{h^2}\frac{\partial E_{zs}}{\partial x} - \frac{\gamma}{h^2}\frac{\partial H_{zs}}{\partial y} \quad \text{----------} \quad [27]$$

where $h^2 = \gamma^2 + k^2 = k_x^2 + k_y^2$

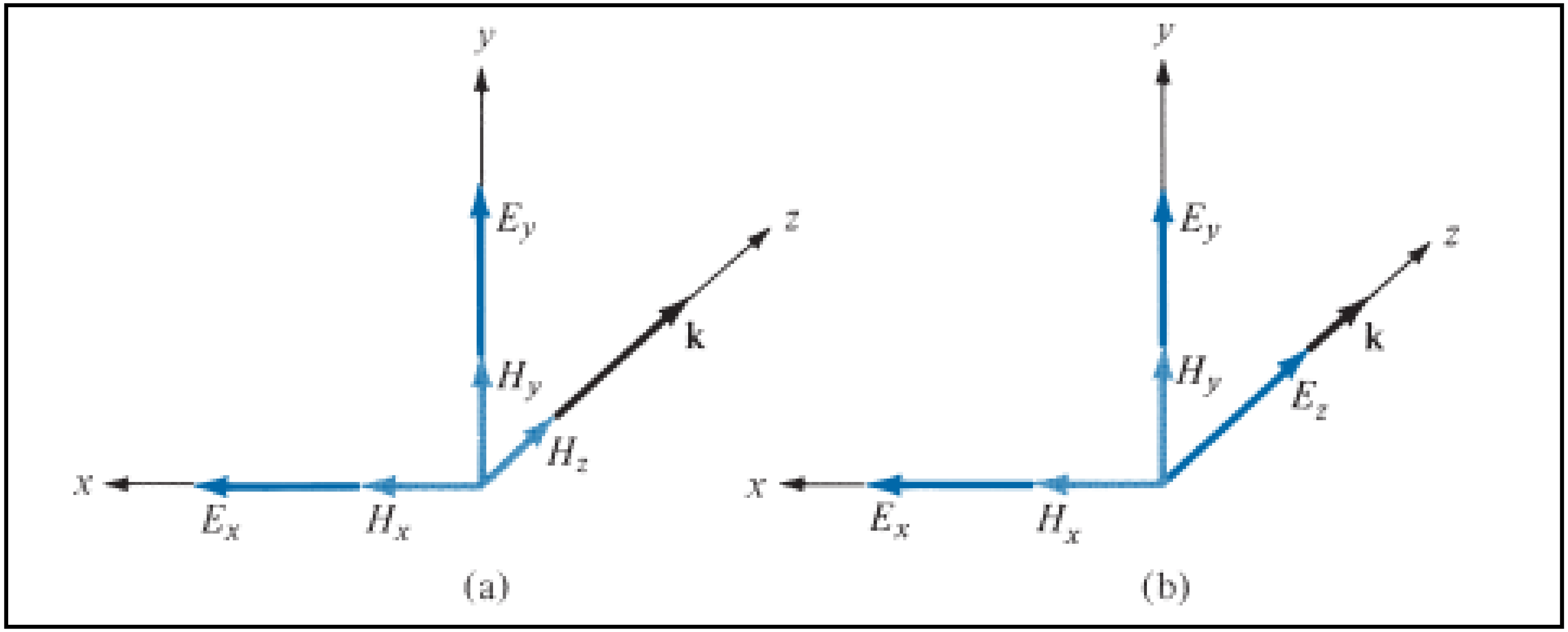

FIGURE 3: Components of EM fields in a rectangular waveguide:(a) TE mode E_z = 0, (b) TM mode, H_z = 0.

- **Thus, final conclusions we have found that from last calculation. These are summarized as follows- From equations (15), (16), and (24-27), we observe that the field patterns or configurations get in different types. Each of these different field patterns is known as a *mode*. There are four different mode categories, these are namely:**

1. $E_{zs} = 0 = H_{zs}$ (TEM mode): In the *transverse electromagnetic* mode, both the **E** and **H** fields are transverse to the direction of wave propagation. From eq **(24-27)** , all field components vanish for $E_{zs} = 0 = H_{zs}$. Consequently, we conclude that a hollow rectangular waveguide cannot support TEM mode.
2. $E_{zs} = 0, H_{zs} \neq 0$ (TE modes): For this case, the remaining components (E_{xs} and E_{ys}) of the electric field are transverse to the direction of propagation $\mathbf{a}_z$. Under this condition, fields are said to be in *transverse electric* (TE) modes. See Figure .3(a).
3. $E_{zs} \neq 0, H_{zs} = 0$ (TM modes): In this case, the **H** field is transverse to the direction of wave propagation. Thus we have *transverse magnetic* (TM) modes. See Figure .3(b).
4. $E_{zs} \neq 0, H_{zs} \neq 0$ (HE modes): In this case neither the **E** nor the **H** field is transverse to the direction of wave propagation. Sometimes these modes are referred to as *hybrid* modes.

Next ----TM and TE MODES will be discussed in next lecture. For any query/ problem contact me on whats app group or mail on my email- arvindkumar@mgcub.ac.in .
##Stay at home. Healthy and safe

References:

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- Engineering Electromagnetics by WH Hayt and J A Buck
- Elements of Electromagnetic Theory & Electrodynamics, Satya Prakash

Thank you