Lecture-III Programme: B. Sc. (Hons.) Physics Semester-VI Course code: PHYS3013
Course Title: Electromagnetic Theory

Rectangular Wave Guide: Transverse Magnetic [TM] Modes

Dr. Arvind Kumar Sharma (Assistant Professor)
Department of Physics, Mahatma Gandhi Central University, Motihari: 845401, Bihar

Now we will discuss here Transverse Magnetic (TM) mode. In this case of the TM mode, the magnetic field has its components transverse to the direction of wave propagation. This suggests that if we put $H_z = 0$ and resolve E_x, E_y, E_z, H_x, and H_y by utilizing eqs. (14) and (24-27 from (previous lecture II)) and the boundary conditions. We shall solve for E_z and afterward resolve next field components from E_z. At the walls (perfect conductors) of the waveguide in Figure 1, the tangential components of the Electric filed E must be continuous; that is

$E_{zs} = 0 \quad \text{at} \quad y = 0$	[Bottom wall]	-------------	[1]
$E_{zs} = 0 \quad \text{at} \quad y = b$	[Top wall]	-------------	[2]
$E_{zs} = 0 \quad \text{at} \quad x = 0$	[Right wall]	-------------	[3]
$E_{zs} = 0 \quad \text{at} \quad x = a$	[Left wall]	-------------	[4]

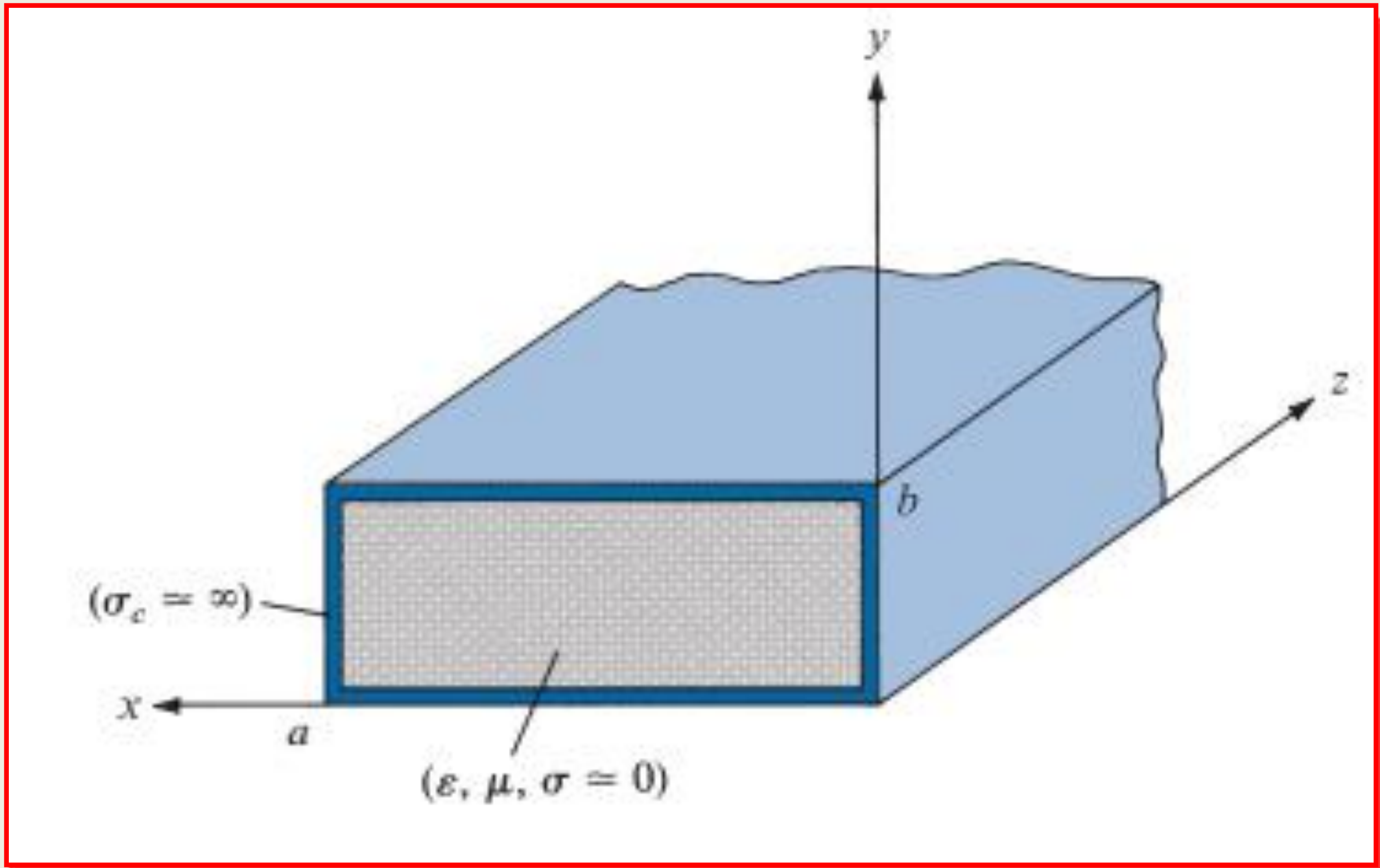

FIGURE 1: A rectangular waveguide with perfectly conducting walls, filled with a lossless material [Ref-1] .

Equations (1) and (3) need that $A_1 = 0 = A_3$ as in eq. (15 (from lecture II)), so this eq. (15 (from lecture II)) turns into

$$E_{zs} = E_0 \sin k_x x \sin k_y y \, e^{-\gamma z} \qquad \text{-------------- [5]}$$

here $E_o = A_2 A_4$. If eqs. (2) and (4) when utilized to eq. (5) require, correspondingly, that

$$\sin k_x a = 0 \text{ and } \sin k_y b = 0 \qquad \text{-------------- [6]}$$

Finally, it suggests that

$$k_x a = m\pi, \quad \text{when } m = 1, 2, 3, \ldots \qquad \text{-------------- [7]}$$

$$k_y b = n\pi, \quad \text{when } n = 1, 2, 3, \ldots \qquad \text{-------------- [8]}$$

$$k_x = \frac{m\pi}{a}, \quad k_y = \frac{n\pi}{b}$$ -------------- [9]

The negative integers are not chosen for *m* and *n* in eqs. (7) and (8) Now putting the values of k_x and k_y from eq. (9) into eq. (5) that gives

$$E_{zs} = E_o \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$ -------------- [10]

Now to find other field components by eqs. (10) and using equations (24-27 (from lecture II)) by setting $H_{zs} = 0$. Thus-

$$E_{xs} = -\frac{\gamma}{h^2}\left(\frac{m\pi}{a}\right) E_o \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z} \quad \text{-----------------} [11]$$

$$E_{ys} = -\frac{\gamma}{h^2}\left(\frac{n\pi}{b}\right) E_o \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z} \quad \text{-----------------} [12]$$

$$H_{xs} = \frac{j\omega\varepsilon}{h^2}\left(\frac{n\pi}{b}\right) E_o \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z} \quad \text{-----------------} [13]$$

$$H_{ys} = -\frac{j\omega\varepsilon}{h^2}\left(\frac{m\pi}{a}\right) E_o \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z} \quad \text{----------------} [14]$$

where

$$h^2 = k_x^2 + k_y^2 = \left[\frac{m\pi}{a}\right]^2 + \left[\frac{n\pi}{b}\right]^2 \quad \text{---------------} [15]$$

which is achieved from eqs. (28 from last lect II) and (9). Observe from eqs. (10-14) that each set of integers *m* and *n* provides a different field configuration or mode, and it is represented by TM_{mn} mode, in the waveguide.

Integer *m* equals the number of half-cycle deviations in the *x*-direction, and integer *n* is the number of half-cycle deviations in the *y*-direction. We also observe from eqs. (10-14) that if (*m*, *n)* is (0, 0), (m, 0), or (0, *n)*, 0 2, every field components disappear. Thus neither *m* nor *n* can be zero. Therefore, TM_{11} is the lowest-order mode from all of the TM_{mn} modes. Now, by putting eq. (9) into eq. (28 from previous lecture II), we get the propagation constant-

$$\gamma = \sqrt{\left[\frac{m\pi}{a}\right]^2 + \left[\frac{n\pi}{b}\right]^2 - k^2}$$ -------------- [16]

where $$k = \omega\sqrt{\mu\varepsilon}$$ -------------- [17]

where $k = \omega\,(\mu\varepsilon)^{1/2}$. We know that, in general, $\gamma = \alpha + i\beta$. In the case of eq. (16), we have three ways depending on k (or ω), m, and n:

CASE I- [Cutoff]

If

$$k^2 = \omega^2\mu\varepsilon = \left[\frac{m\pi}{a}\right]^2 + \left[\frac{n\pi}{b}\right]^2$$

$$\gamma = 0 \quad \text{or} \quad \alpha = 0 = \beta$$

The value of ω that basis this is known as the *cutoff angular frequency* ω_c;

that is,

$$\omega_c = \frac{1}{\sqrt{\mu\varepsilon}}\sqrt{\left[\frac{m\pi}{a}\right]^2 + \left[\frac{n\pi}{b}\right]^2} \qquad \text{--------------- [18]}$$

Propagation does not take place at this frequency

CASE II- [Evanescent]

If

$$k^2 = \omega^2\mu\varepsilon < \left[\frac{m\pi}{a}\right]^2 + \left[\frac{n\pi}{b}\right]^2$$

$$\gamma = \alpha, \qquad \beta = 0$$

In this case, we have no wave propagation at all. These non-propagating modes are said to be *evanescent*.

CASE III [Propagation]

If

$$k^2 = \omega^2\mu\varepsilon > \left[\frac{m\pi}{a}\right]^2 + \left[\frac{n\pi}{b}\right]^2$$

$$\gamma = j\beta, \qquad \alpha = 0$$

that is, from eq. (16) the phase constant β turns into

$$\beta = \sqrt{k^2 - \left[\frac{m\pi}{a}\right]^2 - \left[\frac{n\pi}{b}\right]^2} \qquad \text{-------------- [19]}$$

This is the single case in which propagation takes place, as all field components will have the factor $e^{-\gamma z} = e^{-j\beta z}$. Hence for each mode, characterized by a set of integers *m* and *n*, there is a corresponding *cutoff frequency dentoed by f_c*.

The cutoff frequency is the operating frequency below which attenuation occurs and above which propagation takes place.

❖**The waveguide therefore works as a high-pass filter. The cutoff frequency is achieved from eq. (18) as-**

$$f_c = \frac{\omega_c}{2\pi} = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left[\frac{m\pi}{a}\right]^2 + \left[\frac{n\pi}{b}\right]^2}$$

$$f_c = \frac{u'}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

-------------- [20]

where u' [$u' = \frac{1}{\sqrt{\mu\varepsilon}}$] is representing the phase velocity of uniform plane wave in the lossless dielectric medium (σ = 0, μ, ε) filling the waveguide. The *cutoff wavelength* λ_c is given by

$$\lambda_c = \frac{u'}{f_c}$$

$$\lambda_c = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}} \quad \text{-------------- [21]}$$

It is observed that from eqs. (20) and (21) that TM_{11} has the lowest cutoff frequency (or the longest cutoff wavelength) of all the TM modes. The phase constant β in eq. (19) can be written in terms of f_c as-

$$\beta = \omega\sqrt{\mu\varepsilon}\sqrt{1-\left[\frac{f_c}{f}\right]^2}$$

$$\beta = \beta'\sqrt{1-\left[\frac{f_c}{f}\right]^2}$$

------------- [22]

where β = ω/*u* = *(εμ)*$^{1/2}$ phase constant of uniform plane wave in the dielectric medium. It should be noted that γ for *evanescent* mode can be written in terms of f_c, as-

$$\gamma = \alpha = \beta'\sqrt{\left(\frac{f_c}{f}\right)^2 - 1}$$

------------- [23]

The phase velocity u_p and the wavelength in the waveguide are, respectively, defined as

$$u_p = \frac{\omega}{\beta'}, \lambda = \frac{2\pi}{\beta} = \frac{u_p}{f}$$

The intrinsic wave impedance of the mode is determined by using eqs. (11-14) as

$\gamma = i\beta$

$$\eta_{TM} = \frac{E_x}{H_y} = -\frac{E_y}{H_x} = \frac{\beta}{\omega\varepsilon} = \sqrt{\frac{\mu}{\varepsilon}}\sqrt{1 - \left[\frac{f_c}{f}\right]^2}$$

$$\eta_{TM} = \eta'\sqrt{1 - \left[\frac{f_c}{f}\right]^2}$$

-------------- [24]

Where $\eta = (\mu / \varepsilon)^{1/2}$ is the intrinsic impedance of a uniform plane wave in the medium. Notice that the difference between u', β' , and η' , and u, β, and η, the primed quantities are wave characteristics of the dielectric medium unbounded by the waveguide, as earlier discussed in previous lecture II (i.e., for TEM mode). For instance, u' would be the velocity of the wave if the waveguide were detached and the whole space were filled with the dielectric. The unprimed quantities are the wave characteristics of the medium bounded by the waveguide.

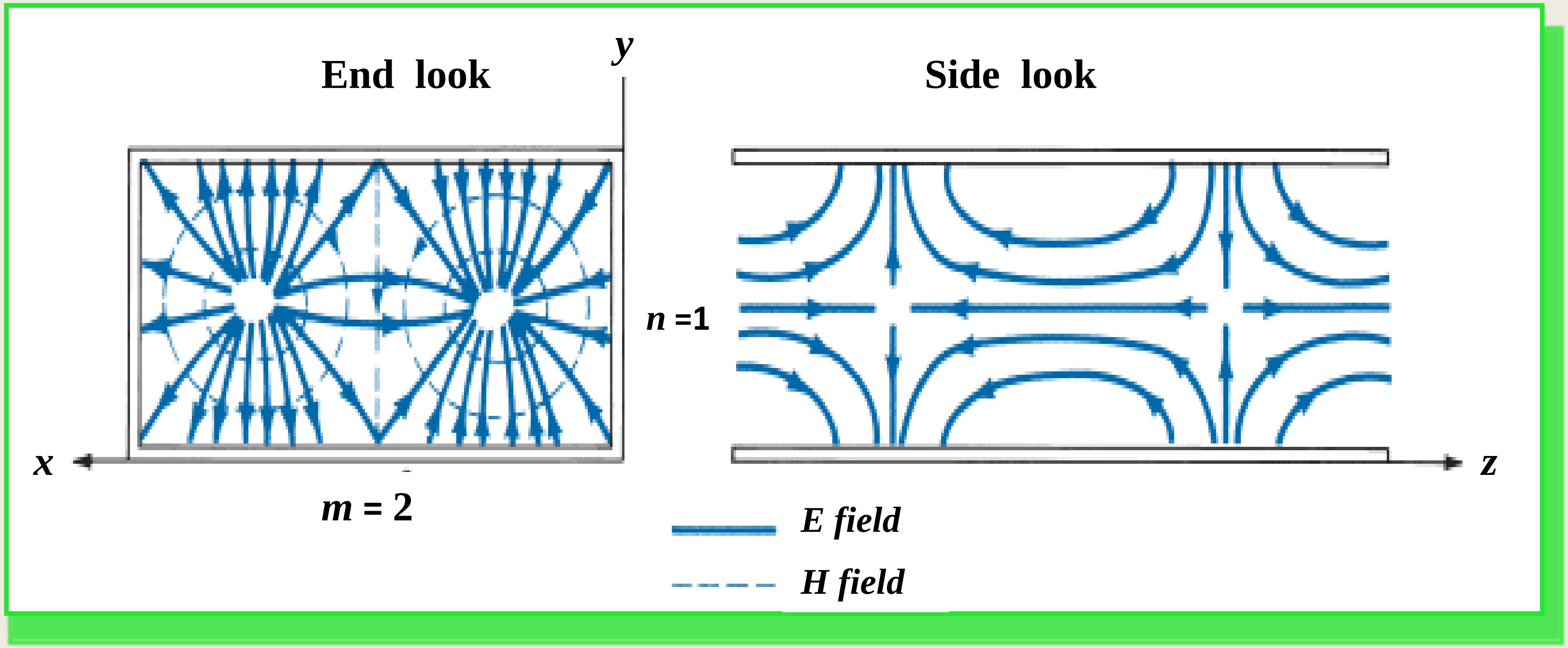

FIGURE 2: Field pattern for TM_{21} mode [Ref. 1]

As pointed out as earlier, the integers *m* and *n* show the number of half-cycle variations in the *x*–*y* cross section of the waveguide. hence for a fixed time, the field pattern of Figure 2 results for TM_{21} mode.

References:

1. **Elements of Electromagnetics, 2nd edition by M N O Sadiku.**

2. **Engineering Electromagnetics by W H Hayt and J A Buck.**

3. **Elements of Electromagnetic Theory & Electrodynamics, Satya Prakash**

- **For any query/ problem contact me on whatsapp group or mail on me**

  E-mail: arvindkumar@mgcub.ac.in

- **Next ----TE Modes will be discussed in next lecture.**

Stay at home. Stay safe and healthy

Thank you