Lecture-IV Programme: M. Sc./B.Sc. Physics

Rectangular Wave Guide: Transverse Electric [TE] Modes

Dr. Arvind Kumar Sharma (Assistant Professor)

Department of Physics, Mahatma Gandhi Central University, Motihari: 845401, Bihar

Now we will discuss here Transverse Electric (TE) modes. In this case of the TE mode, the electric field has its components transverse to the direction of wave propagation. This suggests that if we put E_z = 0 and resolve E_x, E_y, H_x, H_y and H_z by utilizing eqs. (16 and 24-27 from (previous lecture II)) and the boundary conditions. The boundary conditions are taken from the constraint that the tangential components of the electric field be same at the walls (perfect conductors) of the waveguide; these are as -

$$E_{xs} = 0 \quad \text{at} \quad y = 0 \qquad \text{-------------} \quad [1]$$

$$E_{xs} = 0 \quad \text{at} \quad y = b \qquad \text{-------------} \quad [2]$$

$$E_{ys} = 0 \quad \text{at} \quad x = 0 \qquad \text{-------------} \quad [3]$$

$$E_{ys} = 0 \quad \text{at} \quad x = a \qquad \text{-------------} \quad [4]$$

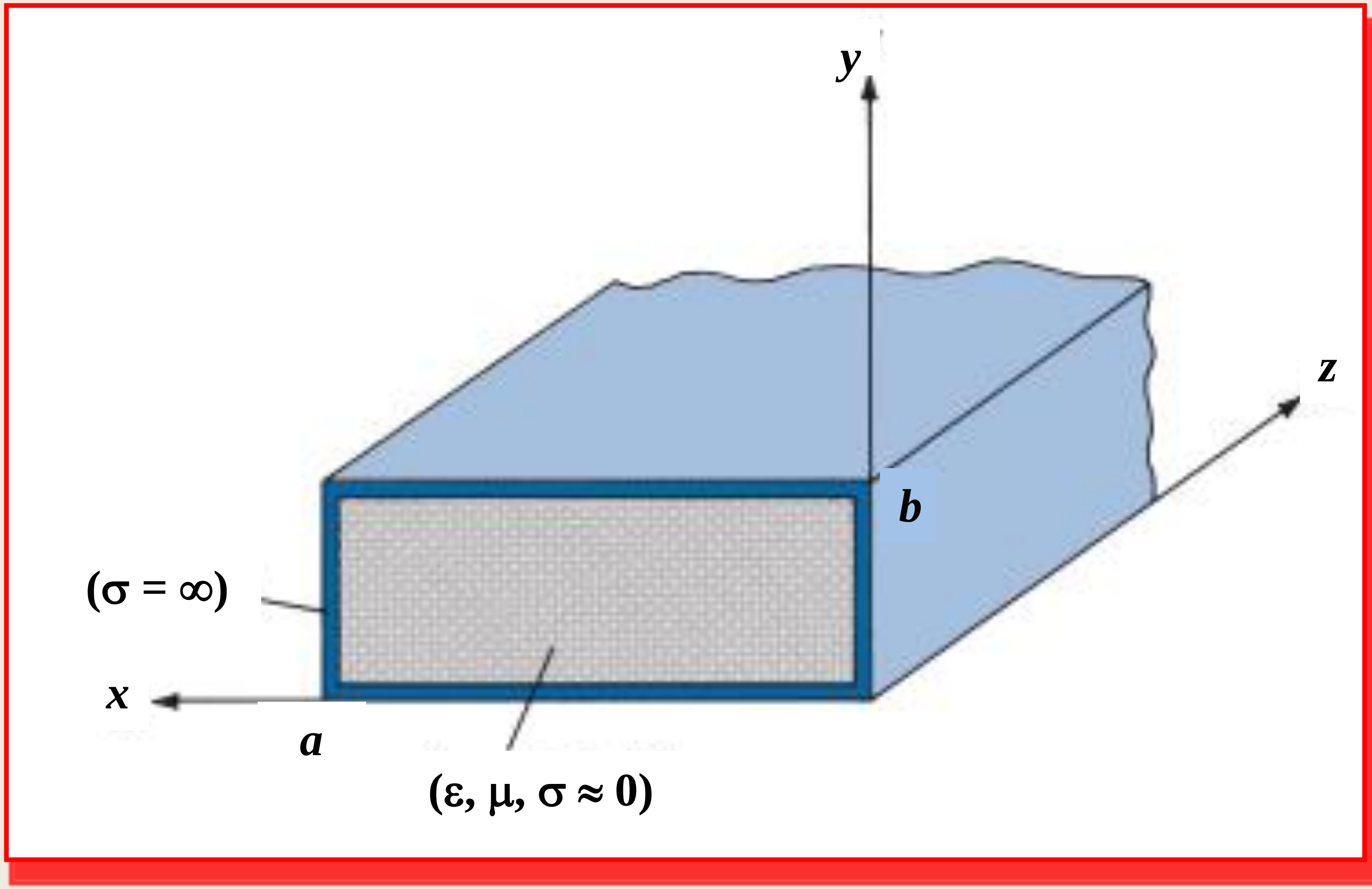

Figure 1: A rectangular waveguide with perfectly conducting walls, filled with a lossless material [*Ref-1] .

From eqs. (16 from previous lect II) and (1-4), the boundary conditions can be expressed as-

$$\frac{\partial H_{zs}}{\partial y} = 0 \quad \text{at} \quad y = 0 \qquad \text{------------ [5]}$$

$$\frac{\partial H_{zs}}{\partial y} = 0 \quad \text{at} \quad y = b \qquad \text{------------ [6]}$$

$$\frac{\partial H_{zs}}{\partial x} = 0 \quad \text{at} \quad x = 0 \qquad \text{------------ [7]}$$

$$\frac{\partial H_{zs}}{\partial x} = 0 \quad \text{at} \quad x = a \qquad \text{------------ [8]}$$

Utilizing these boundary conditions on eq. (16) which provide us a field eq.

$$H_{zs} = H_o \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z} \qquad \text{------------ [9]}$$

where $H_0 = B_1 B_3$. Similarly next field components can easily obtain using the eqs. (24-27 (from previous lect-II)) and (9). These are as-

$$E_{xs} = \frac{j\omega\mu}{h^2}\left(\frac{n\pi}{b}\right) H_o \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z} \quad \text{-------------- [10]}$$

$$E_{ys} = -\frac{j\omega\mu}{h^2}\left(\frac{m\pi}{a}\right) H_o \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z} \quad \text{-------------- [11]}$$

$$H_{xs} = \frac{\gamma}{h^2}\left(\frac{m\pi}{a}\right) H_o \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z} \quad \text{--------------- [12]}$$

$$H_{ys} = \frac{\gamma}{h^2}\left(\frac{n\pi}{b}\right) H_o \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z} \quad \text{-------------- [13]}$$

here *m* = 0, 1, 2, 3, . . . ; and *n* = 0, 1, 2, 3, . . . ; *h* and γ constant those are defined for the TM modes in previous case. Once again, here *m* and *n* denote the number of half-cycle deviations in the *x*–*y* cross section of the waveguide.

For instance let an example of case TE_{32} mode, the field configuration pattern is shown in Figure 2. The cutoff frequency f_c, the cutoff wavelength λ_c, the phase constant β, the phase velocity u_p, and the wavelength λ for TE modes are the similar for TM modes [see eqs. (20 to 23 from previous lect-III)).

In TE modes, (m, n) may be (0, 1) or (1, 0) but it can not be (0, 0). Both m and n cannot be zero at the same instance it is due the fact that the force the field components become zero from eqs. (10-13). This clears that the lowest mode can be TE_{10} or TE_{01} depending on the magnitudes of a and b, the dimensions of the waveguide.

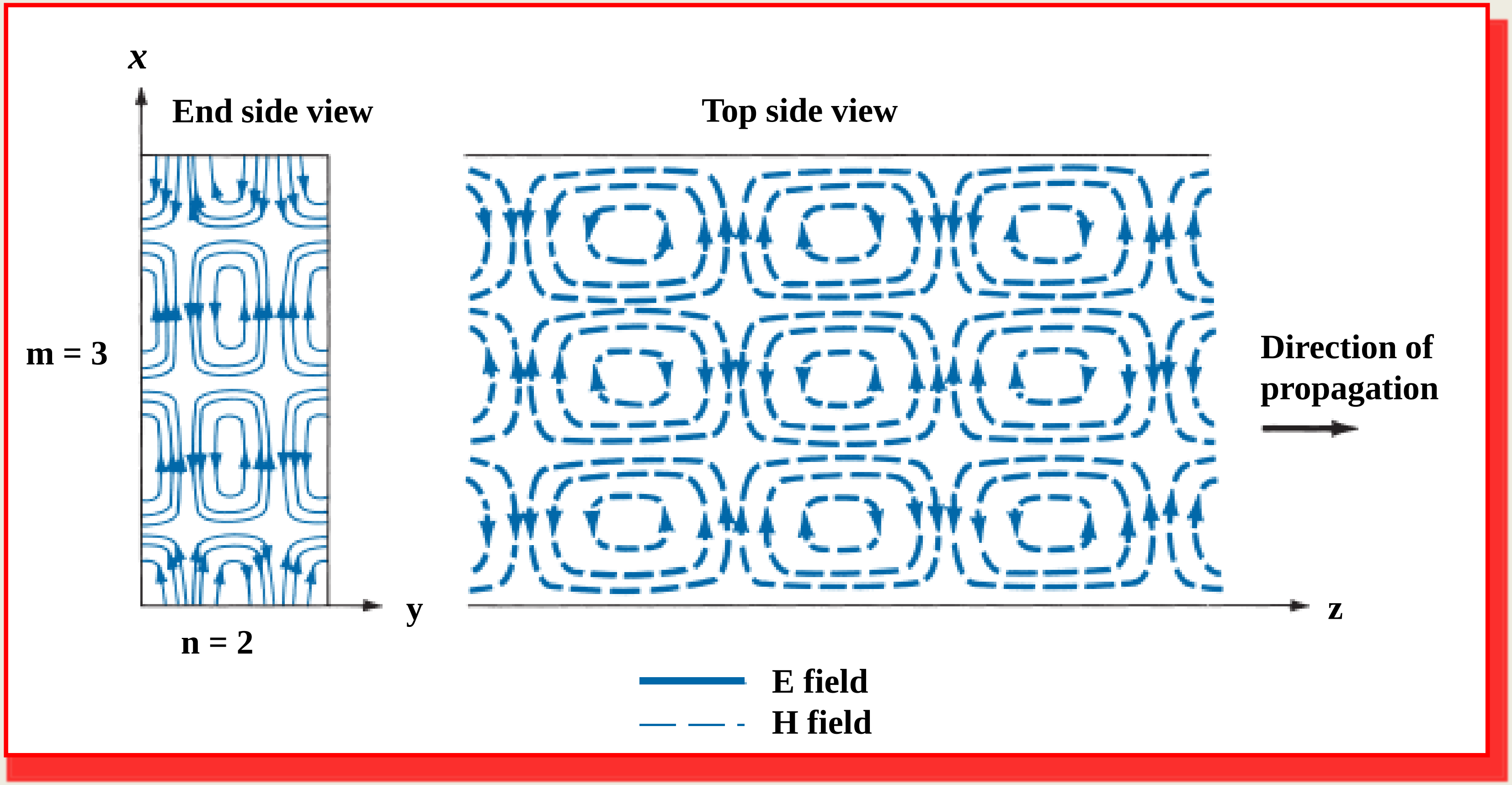

Figure 2: Representation of Field pattern for TE_{32} mode.*[Ref-1]

It is usual practice to have $a > b$ so that $1/a^2 < 1/b^2$ in eq. (20 from previous lect-III). Thus TE_{10} is the lowest mode because $(f_c)_{TE10} = u'/2a < (f_c)_{TE01} = u'/2b$. This mode is called the dominant mode of the waveguide and is of practical importance. The cutoff frequency for the TE10 mode is obtained from eq. (20 from previous lect-III) as (m = 1) , (n = 0)

$$f_{c_{10}} = \frac{u'}{2a} \quad \text{-------------- [14]}$$

and the cutoff wavelength for TE_{10} mode is found using eq. (21 from previous lect-III). It is as-

$$\lambda_{c_{10}} = 2a \qquad \text{-------------- [15]}$$

Note that from eq. (20 from previous lect-III) the cutoff frequency for TM_{11} is

$$\frac{u'[a^2 + b^2]^{1/2}}{2ab} \qquad \text{-------------- [16]}$$

which is larger than the cutoff frequency for TE_{10} mode. Therefore, TM_{11} cannot be viewed as the dominant mode.

The dominant mode is the mode with the lowest cutoff frequency (or longest cutoff wavelength).

❑ It is also note that any EM wave with frequency $f < (f_c)_{10}$ (or $\lambda > (\lambda_c)_{10}$) will not be propagated in the waveguide.

❑ The intrinsic impedance for the TE mode is not similar as for the case discussed in TM mode. From eqs. (10-13), it is obvious that $\gamma = i\beta$. Thus intrinsic impedance is defined as-

$$\eta_{TE} = \frac{E_x}{H_y} = -\frac{E_y}{H_x} = \frac{\omega\mu}{\beta}$$

$$\eta_{TE} = \sqrt{\frac{\mu}{\varepsilon}} \frac{1}{\sqrt{1 - \left[\frac{f_c}{f}\right]^2}}$$

$$\eta_{TE} = \frac{\eta'}{\sqrt{1 - \left[\frac{f_c}{f}\right]^2}}$$

------------- [17]

Note from eqs. (24 from previous lect- III) and (17) that η_{TE} and η_{TM} are purely resistive and vary with frequency, This variation of η_{TE} and η_{TM} are shown in Figure 3. Also notice that from previous calculations-

$$\eta_{TE}\,\eta_{TM} = \eta'^2 \qquad \text{------------- [18]}$$

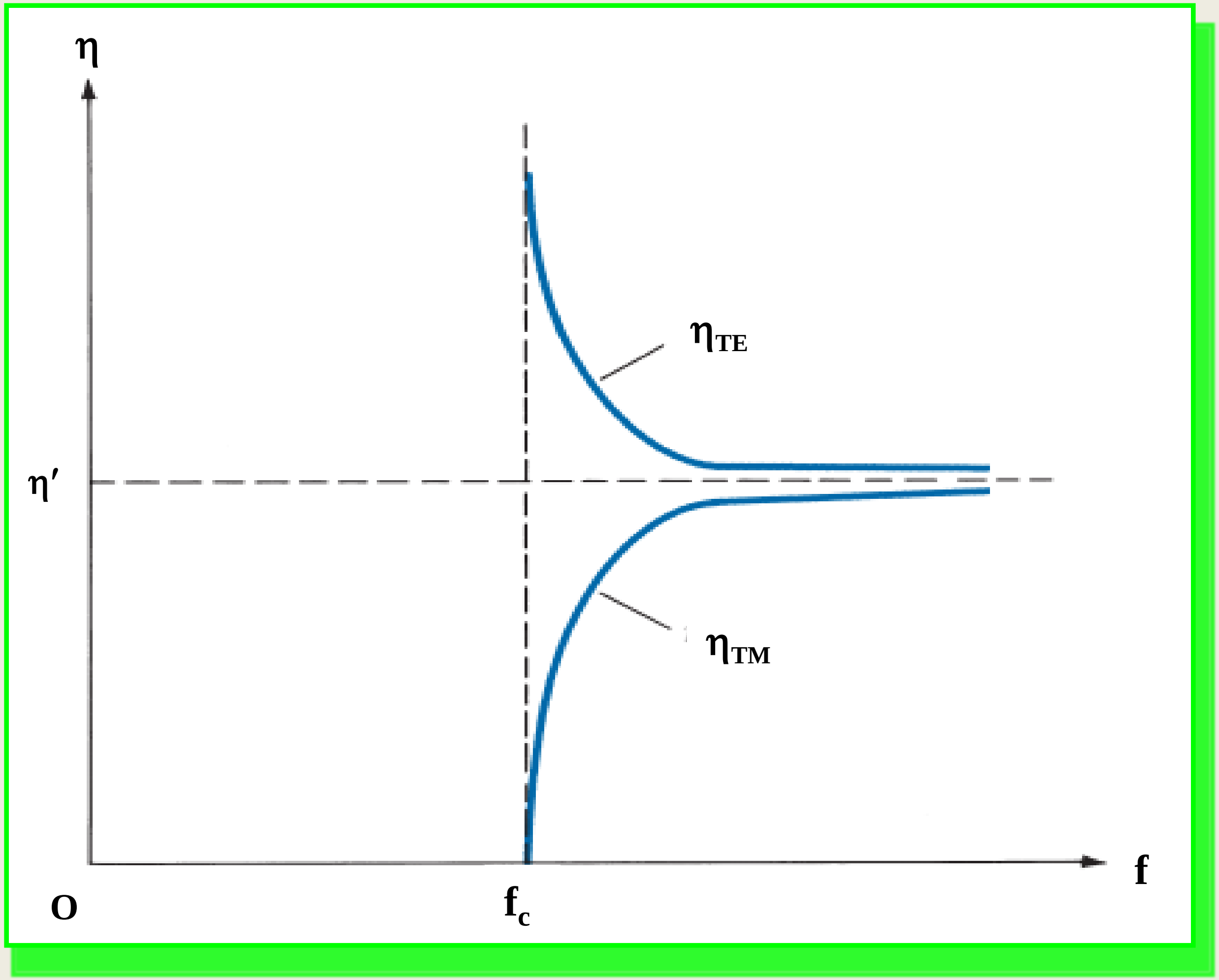

Figure 3:Variation of wave impedance with frequency for TE and TM modes.*[Ref- 1]

From eqs. (10-14 from previous lecture III), equations (9-13), we get the field patterns for the TM and TE modes. For the dominant TE_{10} mode, *m* = 1 and *n* = 0, so eq. (9) turns into-

$$H_{zs} = H_o \cos\left(\frac{\pi x}{a}\right) e^{-j\beta z} \qquad \text{-------------- [19]}$$

The time varying part can be written as-

$$H_z = \mathrm{Re}\left(H_{zs} e^{j\omega t}\right)$$

or

$$H_z = H_o \cos\left(\frac{\pi x}{a}\right) \cos(\omega t - \beta z) \qquad \text{------------- [20]}$$

Similarly, from eqs (10-13) -

$$E_y = \frac{\omega\mu a}{\pi} H_o \sin\left(\frac{\pi x}{a}\right) \sin(\omega t - \beta z) \quad \text{-------------- [21]}$$

$$H_x = -\frac{\beta a}{\pi} H_o \sin\left(\frac{\pi x}{a}\right) \sin(\omega t - \beta z) \quad \text{-------------- [22]}$$

$$E_z = E_x = H_y = 0 \quad \text{-------------- [23]}$$

The variation of the E and H fields with *x* in an *xy* - plane, say plane cos(ωt-βz) = 1 for H_z, and sin(ωt - βz) = 1 for E_y and H_x, is represented in Figure 4 for the TE_{10} mode. The analogous field lines are represented in Figure 5.

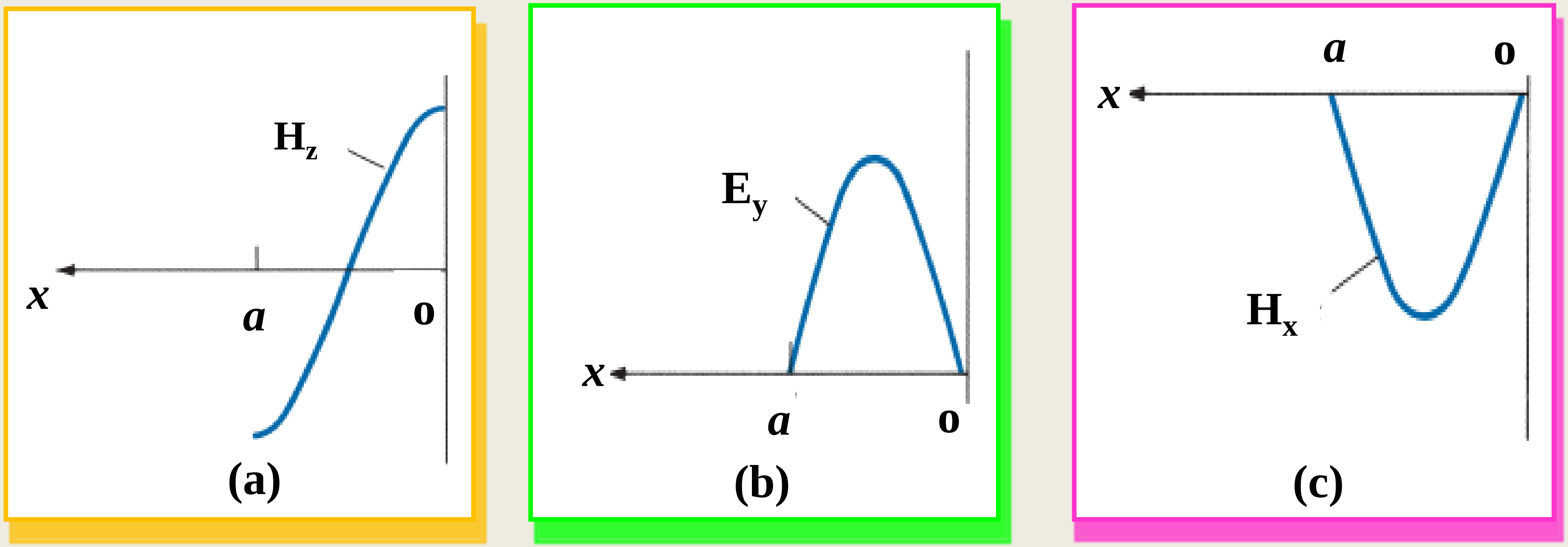

Figure 4: Variation of the field components with x for TE_{10} mode * [Ref-1].

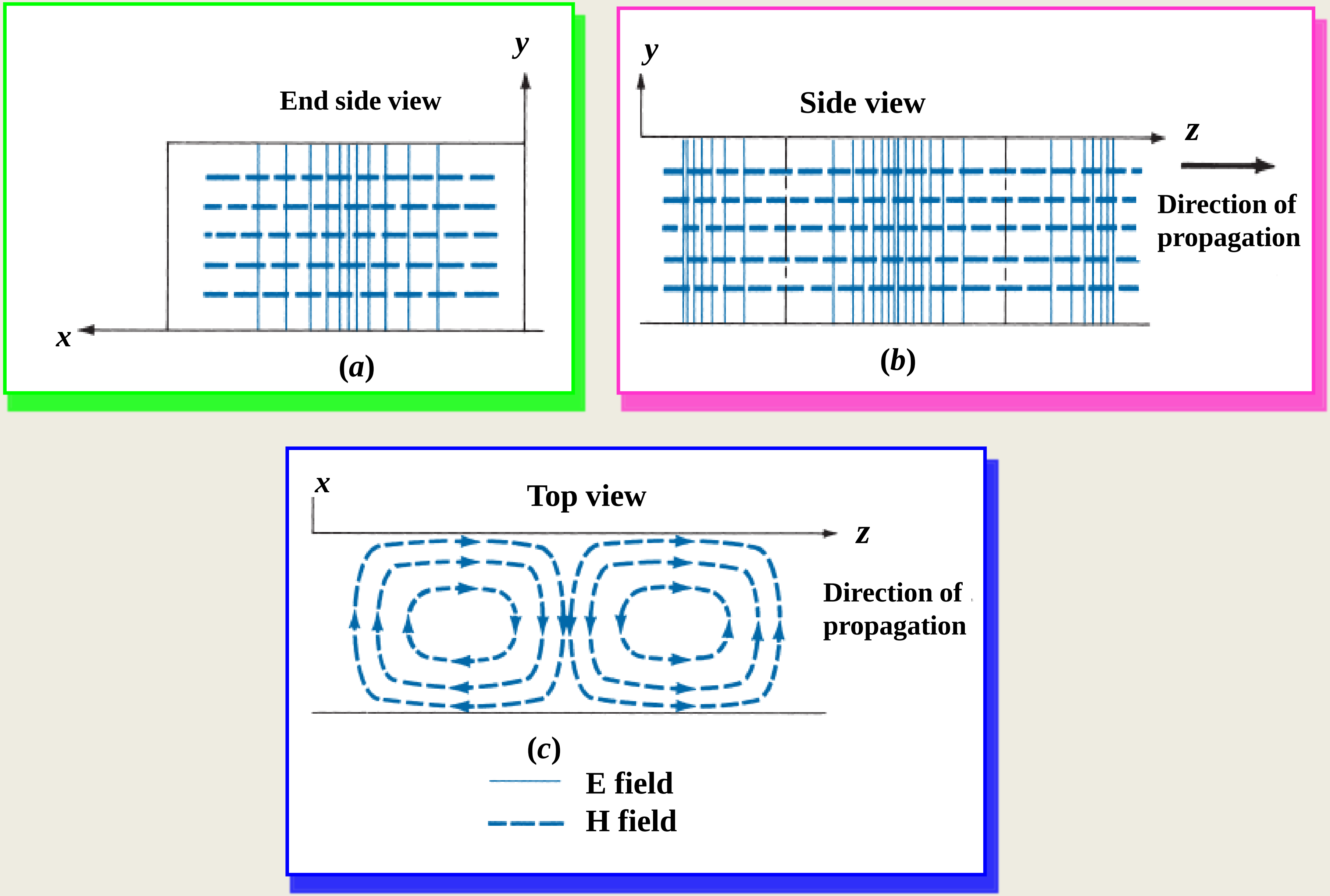

Fig. 5: Field lines for TE_{10} mode, corresponding to components (a), (b), and (c) in Fig. 4 * [Ref-1].

References:

1. Elements of Electromagnetics, 2nd edition by M N O Sadiku.
2. Engineering Electromagnetics by W H Hayt and J A Buck.
3. Elements of Electromagnetic Theory & Electrodynamics, Satya Prakash

- **For any query/ problem contact me on whatsapp group or mail on me**

  E-mail: arvindkumar@mgcub.ac.in

- **Next ***Numerical problems based on these lectures will be discussed in next lecture.**

Stay at home. Stay safe and healthy

Thank you