Course Title: Statistics for Economics
Course Code:ECON4008
Topic: Continuous Probability Distributions
M.A. Economics (2^{nd} Semester)

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Theoretical Distribution

In this topic, we will cover the following *univariate* probability distributions:

i. **Binomial Distribution**
ii. **Poisson Distribution**
iii. **Normal Distribution**

The first two distributions are *discrete* probability distributions and the third is a *continuous* distribution.

Note:

- **Discrete random variable:** Only takes finite or countable many number of values. For example, marks obtained by students in a test, the number of defective mangoes in a basket of mangoes, number of accidents taking place on a busy road, etc.
- **Continuous random variable:** The random variable assume infinite and uncountable set of values. In this case, we usually talk of the value in a particular interval and not at a point. For example, the age, height or weight of students in a class are all continuous random variable.

Normal Distribution

- Normal probability distribution is one of the most important continuous theoretical distributions in Statistics.
- The normal distribution was first discovered in 1733 by English mathematician De-Moivre who obtained this continuous distribution as a limiting case of the binomial distribution and applied it to problems arising in the game of chance.

Definition

- A random variable X is said to have a normal distribution with parameters μ (called "mean") and σ (called "variance") if its density function is given by the probability law:

$$f(x;\mu,\sigma)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left\{\frac{x-\mu}{\sigma}\right\}^2\right]$$

or $$f(x;\mu,\sigma)=\frac{1}{\sigma\sqrt{2\pi}}e^{-(x-\mu)^2/2\sigma^2}$$

$$-\infty<x<\infty,\ -\infty<\mu<\infty,\ \sigma>0$$

- Where π and e are the constants given by: π=22/7. $\sqrt{2\pi}$ =2.5066 and e=2.71828.

Normal Distribution

Standard Normal distribution

- If X is a random variable following normal distribution with mean μ and standard deviation σ, then the random variable Z defined as follows:

$$Z = \frac{X - E(X)}{\sigma_x} = \frac{X - \mu}{\sigma}$$

is called the standard normal variable. We have:

$$E(Z) = E\left(\frac{X-\mu}{\sigma}\right) = \frac{1}{\sigma}E(X-\mu) = \frac{1}{\sigma}[E(X) - E(\mu)] = \frac{1}{\sigma}[\mu - \mu] = 0$$

$$Var(Z) = Var\left(\frac{X-\mu}{\sigma}\right) = \frac{1}{\sigma^2}Var(X-\mu) = \frac{1}{\sigma^2}Var(X) = \frac{1}{\sigma^2}\sigma^2 = 1$$

- Therefore, the standard normal variate Z has mean 0 and standard deviation 1. Hence the probability density function (p.d.f.) of standard normal variate Z is given by:

$$\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}, -\infty < z < \infty$$

Taking x=z, μ=0 and σ=1

Properties of Normal Distribution

- The normal probability curve with mean μ and standard deviation σ is given by the equation

$$f(x) = \frac{1}{\sqrt{2\pi}.\sigma} e^{-(x-\mu)^2/2\sigma^2}, -\infty < x < \infty$$

- The standard normal probability curve is given by the equation:

$$\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}, -\infty < z < \infty$$

- **It has the following properties**

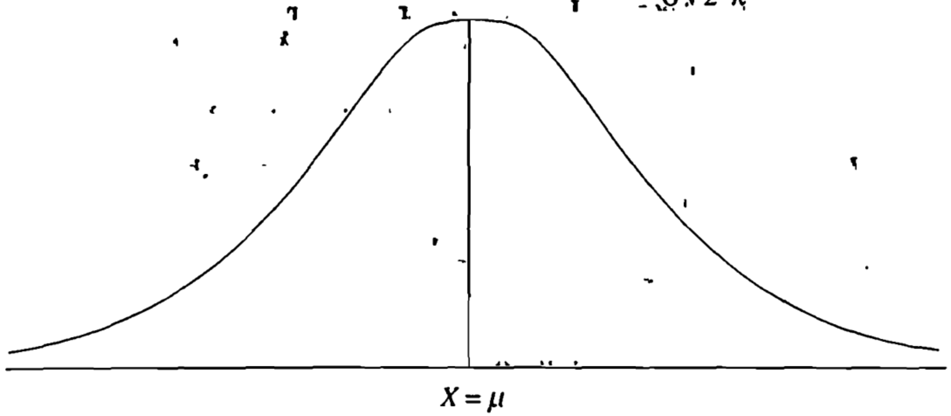
1. The curve is bell shaped and symmetrical about the line x = μ
2. Mean, median and mode of the distribution coincide.
3. As x increases numerically, f(x) decreases rapidly, the maximum probability occurring at the point x = μ, and given by

$$[f(x)]_{\max} = \frac{1}{\sqrt{2\pi}.\sigma}$$

Properties of Normal Distribution

4. If $\beta_1=0$ and $\beta_2=3$
5. $\mu_{2r+1} = 0$, (r = 0, 1,2, ...), and $\mu_{2r} = 1.3.5 \ldots (2r - 1)\, \sigma^{2r}$, (r = 0, 1, 2, ...)
6. Since f(x) being the probability, can never be negative. no portion of the curve lies below the x-axis.
7. Linear combination of independent normal variates is also a normal variate.
8. x-axis is an asymptote to the curve.
9. The points of inflexion of the curve are given by

$$\left[x = \mu \pm \sigma, f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-1/2}\right]$$



(Normal Probability Cuve)

Properties of Normal Distribution

10. Area Property

$$P(\mu - \sigma < X < \mu + \sigma) = 0.6826$$
$$P(\mu - 2\sigma < X < \mu + 2\sigma) = 0.9544$$
$$P(\mu - 3\sigma < X < \mu + 3\sigma) = 0.9973$$

11. The following table gives the area under the normal probability curve for some important values of standard normal variate Z•

Distances from the mean ordinates in terms of $\pm\sigma$	*Area under the curve*
$Z = \pm 0.745$	$50\% = 0.50$
$Z = \pm 1.00$	$68.26\% = 0.6826$
$Z = \pm 1.96$	$95\% = 0.95$
$Z = \pm 2.0$	$95.44\% = 0.9544$
$Z = \pm 2.58$	$99\% = 0.99$
$Z = \pm 3.0$	$99.73\% = 0.9973$

Properties of Normal Distribution

12. If X and Y are independent standard normal variates, then it can be easily proved that U = X + Y and V=X -Y are Independently distributed
13. Area Property (Normal Probability Integral). If X~N(μ , σ^2), then the probability that random value of X will lie between X = μ and X = x_1 is given by

$$P(\mu < X < x_1) = \int_{\mu}^{x_1} f(x)\,dx = \frac{1}{\sigma\sqrt{2\pi}} \int_{\mu}^{x_1} e^{-(x-\mu)^2/(2\sigma^2)}\,dx$$

Put $\frac{X-\mu}{\sigma} = Z$, i.e., $X - \mu = \sigma Z$

When $X = \mu$, $Z = 0$ and when $X = x_1$, $Z = \frac{x_1 - \mu}{\sigma} = z_1$, (say).

$$\therefore \quad P(\mu < X < x_1) = P(0 < Z < z_1) = \frac{1}{\sqrt{2\pi}} \int_0^{z_1} e^{-z^2/2}\,dz = \int_0^{z_1} \varphi(z)\,dz$$

where $\varphi(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$, is the probability function of standard normal variate.

Properties of Normal Distribution

14. The total area under normal probability curve is unity, *i.e.*,

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{\infty} \varphi(z)\,dz = 1$$

Reference:

Gupta, S. C. (2015), *Fundamentals of Statistics*, Himalaya Publishing House.