Flow Networks Part-I (DAA, M.Tech + Ph.D.)

By:

Sunil Kumar Singh, PhD
Assistant Professor,
Department of Computer Science and Information Technology

School of Computational Sciences, Information and Communication Technology,
Mahatma Gandhi Central University, Motihari
Bihar, India-845401

Outline

- Network flow problems
- Max-flow minimum cut
- Ford-Fulkerson algorithm
- Conclusion
- References

Network Flow Problem

- It is a type of network optimization problem.
- General Characteristics
  - ✓ Source: material are produced at a steady rate
  - ✓ Sink: materials are consumed at the same rate as it is being produced from the source.
  - ✓ Flows through conduits are constrained to max values.
- Application areas
  - ✓ Networks: routing as many packets as possible on a given network
  - ✓ Transportation: sending as many trucks as possible, where roads have limits on the number of trucks per unit time
  - ✓ Bridges: destroying some bridges to disconnect s from t, while minimizing the cost of destroying the bridges.

Problem definition and Constraints

- Maximizing the total amount of flow from s to t subject to that does not violate any constraints.
- Given a directed graph G=(V,E), where each edge e is associated with its capacity c(e)>0 for its two source nodes source s and sink t.
- The flow f:VxV →R satisfies the following constraints.

Capacity constraint: For all $u, v \in V$, we require $f(u, v) \le c(u, v)$.

Skew symmetry: For all $u, v \in V$, we require $f(u, v) = -f(v, u)$.

Flow conservation: For all $u \in V - \{s, t\}$, we require

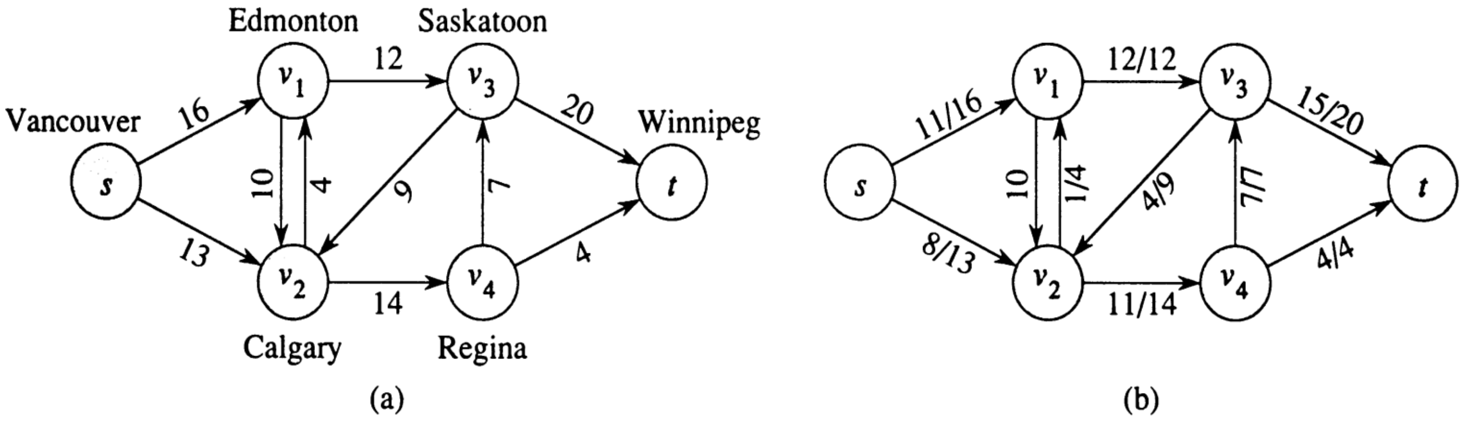
$$\sum_{v \in V} f(u, v) = 0 .$$

The quantity $f(u, v)$, which can be positive or negative, is called the ***net flow*** from vertex u to vertex v. The ***value*** of a flow f is defined as

$$|f| = \sum_{v \in V} f(s, v) , \tag{27.1}$$

Cont..

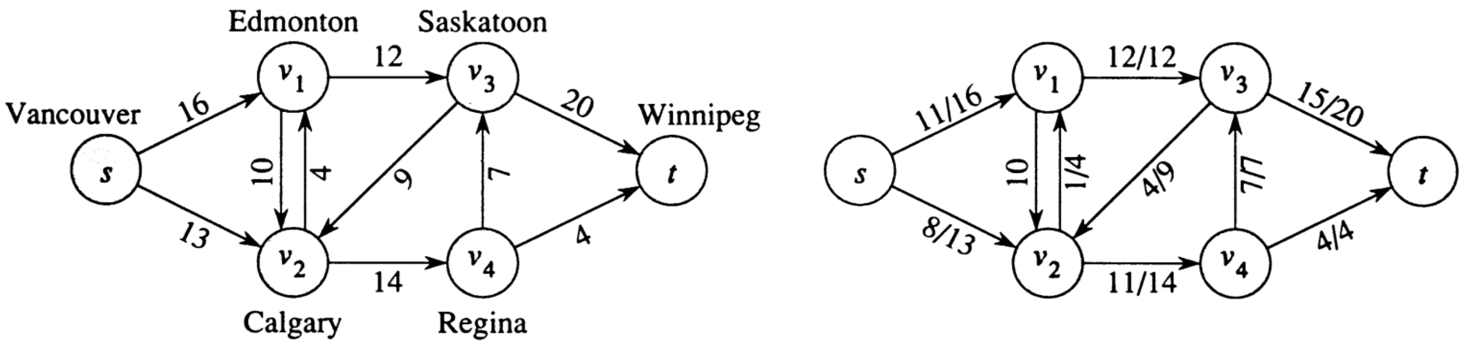
- There is no net flow between vertices u and v, if there is no edge between them.
- **Flow networks example:**



- Figure (a) shows the maximum shipping capacities from the source Vancouver (s) to Winnipeg (t)
- In figure (b), a flow f in G with value |f| = 19 one possible flow is shown

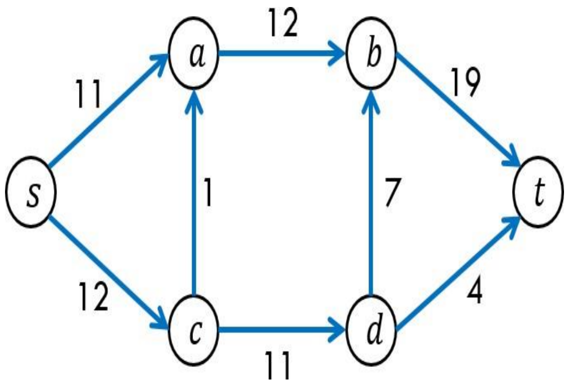
- In which, it turns out that 19 is not the maximum flow, then find out the maximum flow.

- Find the maximum flow



(a) (b)

- Solution is 23 units



Alternate method: for Maximum Flow

- We remove some edges from the graph such that after removing the edges, there is no path from s to t.

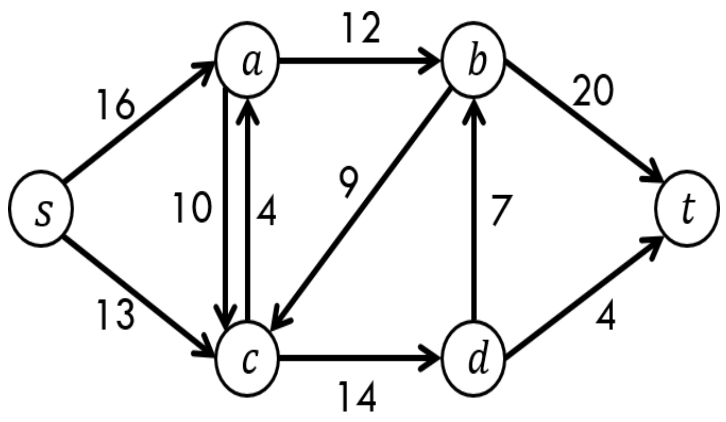
- The cost of removing e is equal to capacity c(e)

- The minimum cut problem is to find a cut with minimum total cost

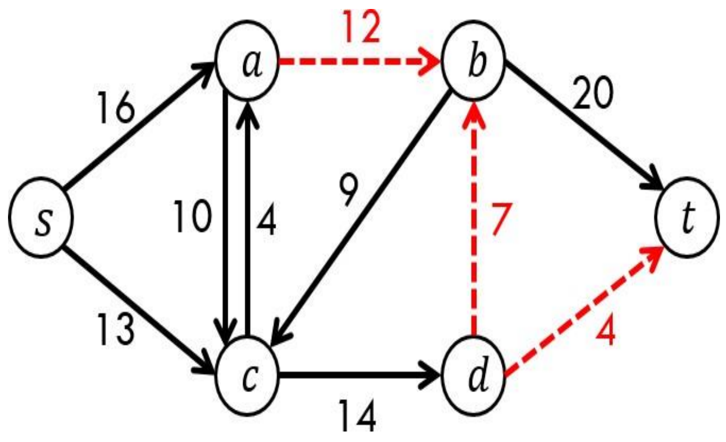
- Theorem: Maximum flow=minimum cut

Minimum cut example

- Capacities mentioned on the link.

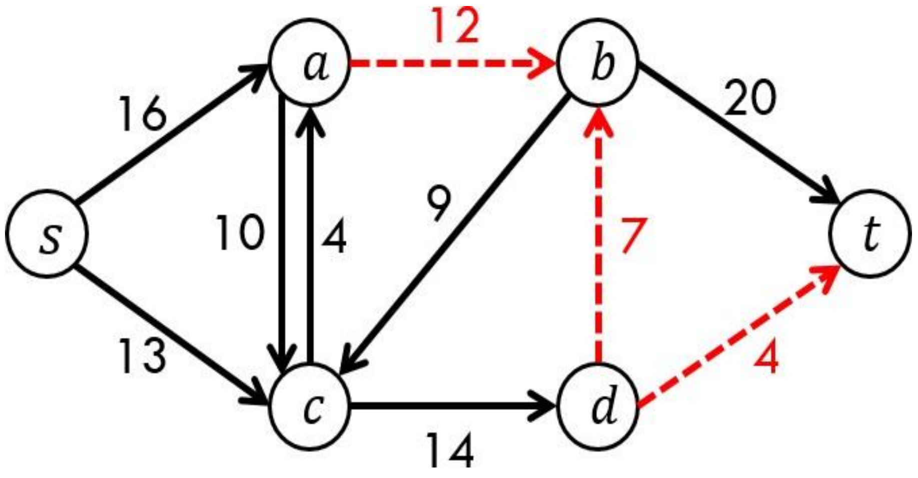


- Minimum Cut (red edges are removed)



Minimum cut example

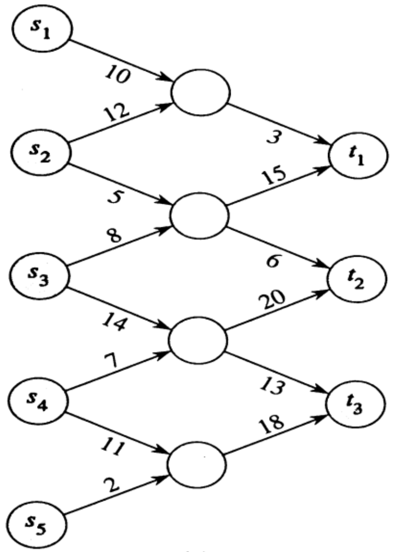
- An valid flow can be decomposed into flow paths and circulations



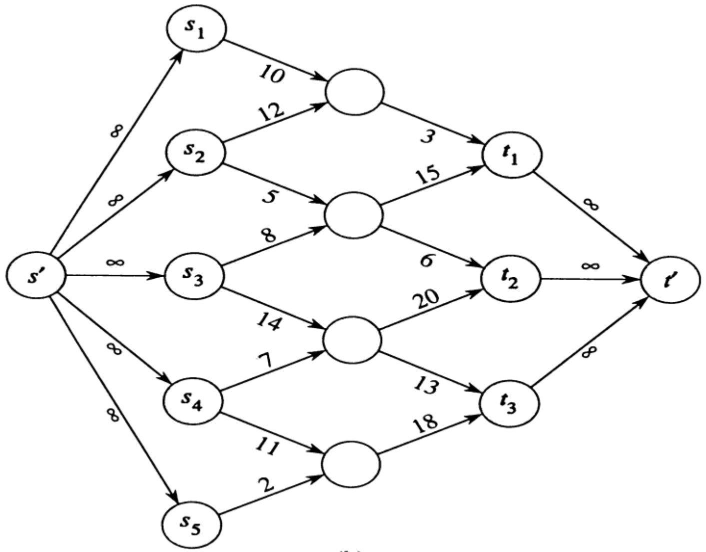
  - $s \rightarrow a \rightarrow b \rightarrow t$:11
  - $s \rightarrow c \rightarrow a \rightarrow b \rightarrow t$:1
  - $s \rightarrow c \rightarrow d \rightarrow b \rightarrow t$:7
  - $s \rightarrow c \rightarrow d \rightarrow t$: 4

Multiple Sources and Sinks

- Problem with multiple sources and sinks can be reduced to the single source/sink case
- A supersource with ∞ outgoing capacities to the multiple sources is added



(a)



(b)

- A supersink with ∞ incoming capacities from the multiple sinks is added

References

1. Cormen, Thomas H., Charles E. Leiserson, Ronald L. Rivest, and Clifford Stein. *Introduction to algorithms*. MIT press, 2009.
2. Cormen, Thomas H., Charles E. Leiserson, Ronald L. Rivest, and Clifford Stein. "Introduction to algorithms second edition." *The Knuth-Morris-Pratt Algorithm, year* (2001).
3. Seaver, Nick. "Knowing algorithms." (2014): 1441587647177.

Thank You