Course Title: Statistics for Economics
Course Code:ECON4008
Topic:Testing of Hypothesis
M.A. Economics (2nd Semester)

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Testing of Hypothesis

- The theory of testing of hypothesis initiated by J. Neyman and E.S. Pearson.
- In Neyman-Pearson theory, the statistical techniques are used to arrive at decisions in certain situations where there is uncertainty on the basis of a sample whose size is fixed in advance.

Statistical Hypothesis - Simple and Composite

- A statistical hypothesis is some statement or assumption about a population or equivalently about the probability distribution characterising a population which we want to verify on the basis of information available from a random sample.
- If the statistical hypothesis specifies the population completely then it is termed as a simple statistical hypothesis. otherwise it is called a composite statistical hypothesis.
- For example, if $X_1, X_2, \ldots, X_n$ is a random sample of size n from a normal population with mean μ and variance σ^2 then the hypothesis

$$H_0 : \mu = \mu_0, \sigma^2 = \sigma_0^2$$

is a simple hypothesis. It completely specifies the distribution.

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- The following hypotheses is a composite hypothesis:

  (i) $\mu = \mu_0$, (ii) $\sigma^2 = \sigma_0^2$

  (iii) $\mu < \mu_0, \sigma^2 = \sigma_0^2$ (iv) $\mu > \mu_0, \sigma^2 = \sigma_0^2$

  (v) $\mu = \mu_0, \sigma^2 < \sigma_0^2$, (vi) $\mu = \mu_0, \sigma^2 > \sigma_0^2$

  (vii) $\mu < \mu_0, \sigma^2 > \sigma_0^2$.

Test of a Statistical Hypothesis.

- A test of a statistical hypothesis is a two-action decision problem after the experimental sample values have been obtained. the two-actions being the acceptance or rejection of the hypothesis under consideration.

Null Hypothesis.

- The technique of randomisation used for the selection of sample units makes the test of significance valid for us. For applying the test of significance we first set up a **hypothesis - *a definite statement about the population parameter***. Such a hypothesis, which is ***usually a hypothesis of no difference, is called null hypothesis*** and is usually denoted by H_0. According to Prof. RA. Fisher, ***null hypothesis is the hypothesis which is tested for possible rejection under the assumption that it is true.***

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Alternative Hypothesis.

- Any hypothesis which is complementary to the null hypothesis is called an alternative hypothesis, usually denoted by H_I. For example, if we want to test the null hypothesis that the population has a specified mean μ_0, i.e., Ho: $\mu = \mu_0$, then the

  (i) $H_1 : \mu \neq \mu_0$ (*i.e.*, $\mu > \mu_0$ or $\mu < \mu_0$)

  (ii) $H_1 : \mu > \mu_0$

  (iii) $H_1 : \mu < \mu_0$

- The alternative hypothesis in (i) is known as a two tailed alternative and the alternatives in (ii) and (iii) are known as right-tailed and left-tailed alternatives respectively. The setting of alternative hypothesis is very important since it enables us to decide whether we have to use a single-tailed (right or left) or two-tailed test.

Types of Errors in Testing of Hypothesis.

- The main objective in sampling theory is to draw valid inferences about the population parameters on the basis, of the sample results. In practice, we decide to accept or reject the hypothesis after examining a sample from it.

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- In the test procedure, we have the following two types of errors:

  Type I Error : Reject H_o when it is true.

  Type II Error: Accept H_o when it is wrong, i.e., accept H_o when H_I is true.

  P(Reject H_o when it is true} =P(Reject H_o I H_o) =α}

  P {Accept H_o when it is wrong} = P{Accept H_o I H_I=β}

Critical Region.

- A region in the sample space S which amounts to rejection of H_0 is termed as critical region or region of rejection: If ω is the critical region and if t = t(X_I, X_2,…….,X_n) is the value of the statistic based on a random sample of size n, then

$$P\,(t \in \omega \mid H_0) = \alpha,\ P\,(t \in \bar{\omega} \mid H_1) = \beta$$

where $\bar{\omega}$, the complementary set of ω, is called the *acceptance region*.

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Level of Significance.

- The probability 'α' that a random value of the statistic t belongs to the critical region is known as the level of significance. In other words, level of significance is the size of the type I error .
- The levels of significance usually employed in testing of hypothesis are 5% and 1 %.
- If we adopt 5% level of significance, it implies that in 5 samples out of 100, we are likely to reject a correct H_0. In other words, this implies that we are 95% confident that our decision to reject H_0 is correct.
- The level of significance is always fixed in advance before collecting the sample information.

One tailed and Two tailed Tests.

- In any test, the critical region is represented by a portion of the area under the probability curve of the sampling distribution of the test statistic.
- A test of any statistical hypothesis when the alternative hypothesis is one tailed (right tailed or left tailed) is called a one tailed test. For example, a test for testing the mean of a population

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$H_0: \mu=\mu_0$

against the alternative hypothesis:

$H_1: \mu>\mu_0$ (Right tailed) or $H_1: \mu<\mu_0$ (Left tailed) is a single tailed test.

- In the right tailed test, the critical region lies entirely in the right tail of the sampling distribution of X, while for the left tail test, the critical region is entirely in the left tail of the distribution.
- A test of statistical hypothesis where the alternative hypothesis is two tailed such as: $H_0: \mu=\mu_0$ against the alternative hypothesis: $H_1: \mu\neq\mu_0$ ($\mu>\mu_0$ and $\mu<\mu_0$) is known as two tailed test and in such a case the critical region is given by the portion of the area lying in both the tails of the probability curve of the test statistic.
- In a particular problem, whether one tailed or two tailed test is to be applied depends entirely on the nature of the alternative hypothesis. If the alternative hypothesis is two-tailed we apply two-tailed test and if alternative hypothesis is one-tailed, we apply one tailed test.

Critical Values or Significant Values.

- The value of test statistic which separates the critical (or rejection) region and the acceptance region is called the critical value or significant value. It depends upon:

(I) The level of significance used, and

(ii) The alternative hypothesis, whether it is two-tailed or single-tailed.

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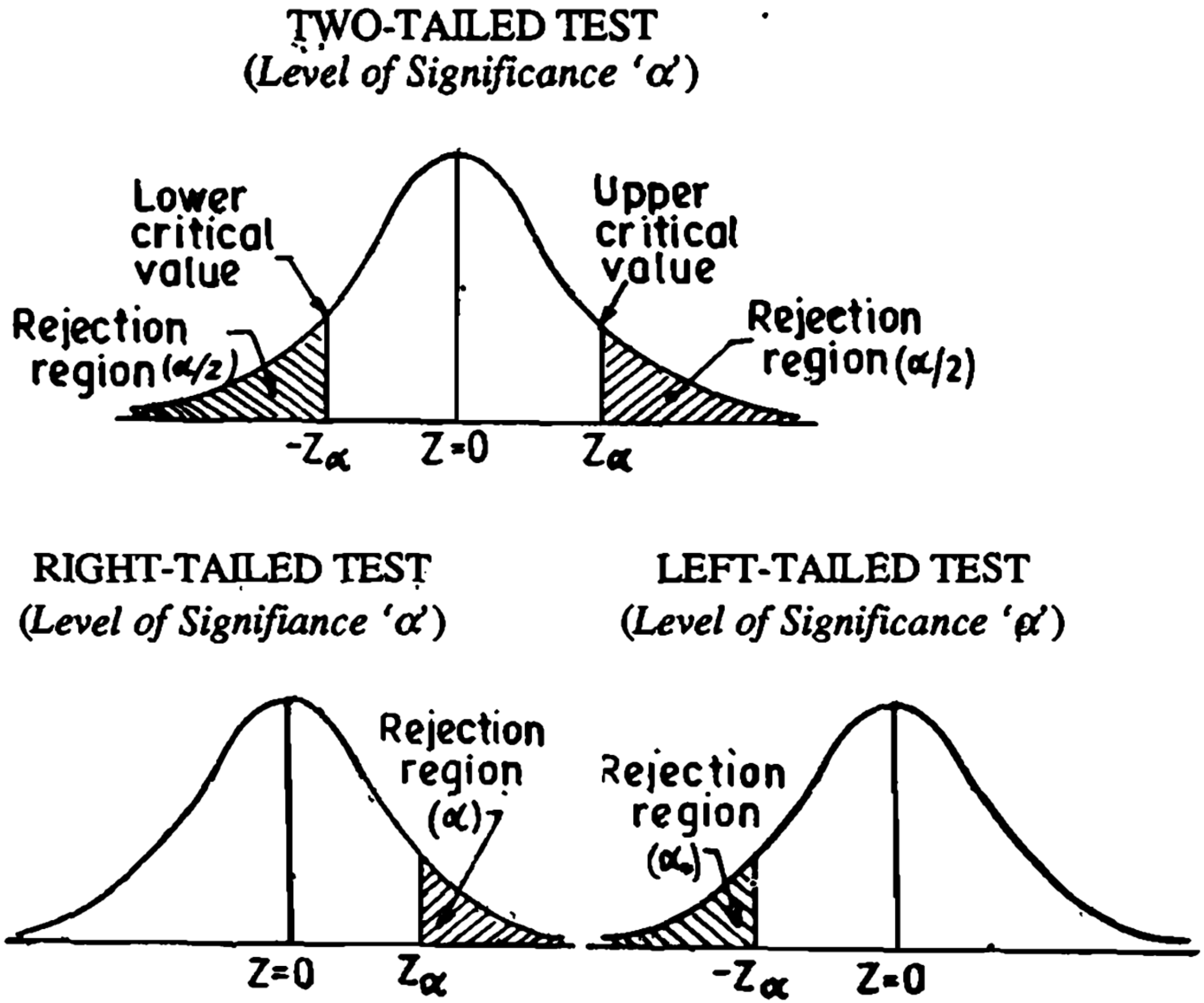

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Procedure for Testing of Hypothesis.

- We now summarise below the various steps in testing of a statistical hypothesis in a systematic manner.

1. Null Hypothesis. Set up the Null Hypothesis H_o.
2. Alternative Hypothesis. Set up the Alternative Hypothesis H_1. This will enable us to decide whether we have to use a single-tailed (right or left) test or two-tailed test.
3. Level of Significance. Choose the appropriate level of Significance (α) depending on the reliability of the estimates and permissible risk. This is to be decided before sample is drawn, i.e. α is fixed in advance.
4. Test Statistic (or Test Criterion). Compute the test statistic under the null hypothesis

$$Z = \frac{t - E(t)}{S.E.(t)}$$

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5. Conclusion. We compare z the computed value of Z in step 4 with the significant value (tabulated value) z_α at the given level of significance, α.
   - If $|Z| < z_\alpha$, i.e if the calculated value of Z (in modulus value) is less than z_α we say it is not significant. In this, the null hypothesis will be accepted.
   - If $|Z| > z_\alpha$ i.e.. if the computed value of test statistic is greater than the critical or Significant value. then we say that it is significant and the null hypothesis is rejected at level of significance α i.e.. with confidence coefficient $(1- \alpha)$.

Reference: Gupta, S. C. (2015), *Fundamentals of Statistics*, Himalaya Publishing House.