B.Sc. (Hons.) Biotechnology
Core Course 13:
Basics of Bioinformatics and Biostatistics (BIOT 3013)

Unit 3:

Test of significance: Z-test, t-test and Chi-square test

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Test of significance

- It is used in estimating population parameters using sample data.
- For example, an administrator of a big hospital is interested in the knowing the mean age of patients admitted during the last year.
- The administrator draw a random sample of size n from the patient population and compute the average $\bar{x}$, which he use as a point estimate of μ.

Test of significance

- Because random sampling involves chance, then it can't be expected to be equal to μ.
- The value of may $\bar{x}$ be greater than or less than μ.
- Statistical inference (test of significance) is the method by which we can reach to a conclusion about a population on the basis of the sample information drawn from the same population.

Hypotheses testing
about population parameters using sample statistics

- It is a statement about one or more populations .
- For example, a hospital administrator may want to test the hypothesis that the average length of stay of patients admitted to the hospital is 5 days.

Hypothesis testing

- There are two hypotheses involved in hypothesis testing

1. **Null hypothesis** H_0: It is the hypothesis to be tested .
2. **Alternative hypothesis** H_A : It is a statement of what we believe is true if our sample data provided reason to reject the null hypothesis.

Hypothesis Testing steps about the mean of a population

1. **Data collection**: find out the determine variable, sample size (n), sample mean($\bar{x}$), population standard deviation (σ) or sample standard deviation (s) if they are unknown .
2. **Assumptions :** Now we have two cases:

- Case1: Population is normally distributed with known or unknown variance (sample size n may be small or large),
- Case2: Population is not normal with known or unknown variance (n is large i.e. n≥30).

- 3.Hypotheses: we have to test three cases
- Case I: we want to test that the population mean is different than 50.

H_0: $\mu=\mu_0$

H_A: $\mu \neq \mu_0$

- Case II : we want to test that the population mean is greater than 50.

H_0: $\mu = \mu_0$

H_A: $\mu > \mu_0$

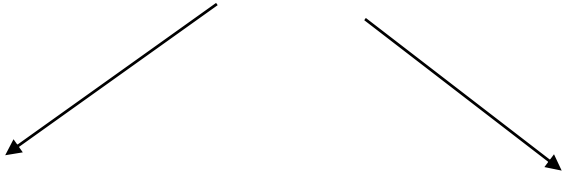
- Case III : we want to test that the population mean is less than 50.

$H_{0:}$ $\mu = \mu_0$

H_A: $\mu < \mu_0$

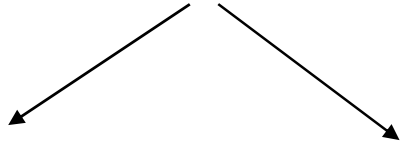
4.Test Statistic:

- **Case 1:** Population is normal distributed.



σ^2 is known (n large or small)

σ^2 is unknown



n large

n small

$$Z = \frac{\overline{X} - \mu_o}{\sigma / \sqrt{n}}$$

$$Z = \frac{\overline{X} - \mu_o}{s / \sqrt{n}}$$

$$t = \frac{\overline{X} - \mu_o}{s / \sqrt{n}}$$

- Case2: If population is not normal and n is large.

i)If σ^2 is known

$$Z = \frac{\overline{X} - \mu_o}{\sigma / \sqrt{n}}$$

ii) If σ^2 is unknown

$$Z = \frac{\overline{X} - \mu_o}{s / \sqrt{n}}$$

5.Decision Rule on the basis of level of significance (α)

i) If $H_A: \mu \neq \mu_0$

Reject H_0 if $Z > Z_{1-\alpha/2}$ or $Z < -Z_{1-\alpha/2}$ (when use Z - test)

or Reject H_0 if $T > t_{1-\alpha/2,n-1}$ or $T < -t_{1-\alpha/2,n-1}$ (when use t-test)

ii) If $H_A: \mu > \mu_0$

Reject H_0 if $Z > Z_{1-\alpha}$ (when use Z - test)

or Reject H_0 if $T > t_{1-\alpha,n-1}$ (when use t - test)

iii) If $H_A: \mu < \mu_0$

Reject H_0 if $Z < -Z_{1-\alpha}$ (when use Z - test)

Or Reject H_0 if $T < -t_{1-\alpha,n-1}$ (when use t - test)

6. Decision: If we reject H_0, we can conclude that H_A is true.

7. An alternative decision rule can be applied using the p- value.

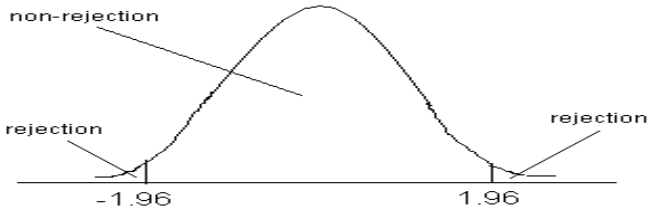
i) If the p-value is less than or equal to α ,we reject the null hypothesis ($p \leq \alpha$).

ii) If the p-value is greater than α ,we do not reject the null hypothesis ($p > \alpha$).

Example 1

- Suppose a researcher is interested in the mean age of a certain population. A random sample of 10 individuals drawn from the target population that has a mean of 27 years. Assuming the population is normally distributed with variance of 20. Can we conclude that the mean is different from 30 years? (α=0.05) . If the p-value is 0.0340, how can we use it in making a decision?

Solution



1-Data: variable is age, n=10,

$\overline{X}$=27,σ^2=20, α=0.05

2-Hypotheses: H_0 : μ=30; H_A: $\mu \neq 30$

3-Test Statistic:

$$Z = \frac{\overline{X} - \mu_o}{\sigma/\sqrt{n}} = \frac{27 - 30}{\sqrt{\frac{20}{10}}}$$

Z calculated = -2.12

4.Decision Rule: The alternative hypothesis is true

H_A: $\mu \neq 30$; hence we reject H_0.

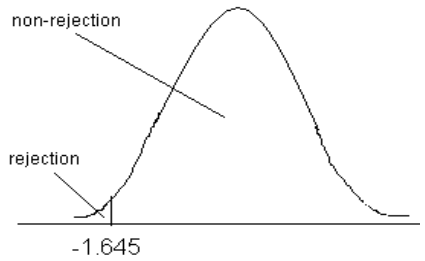
if Z cal. $> Z_{1-0.025/2} = Z_{0.975}$

or Z cal. $< -Z_{1-0.025/2} = -Z_{0.975}$

- $Z_{0.975}$=1.96(from standard table A1)

Can we conclude that μ<30

Decision Rule: Reject H_0 if $Z < Z_{\alpha}$, where $Z_{\alpha} = -1.645$ (from table A1 at $\alpha = 5\%$).



Decision: Thus, we can conclude that the population mean is smaller than 30.

Example 2

- Among 157 African-American men, the sample mean systolic blood pressure was 146 mm Hg with a standard deviation of 27. Assuming the population distribution is not normal, can we conclude that the mean systolic blood pressure for a population of African-American is greater than 140 mm Hg at $\alpha=0.01$.

Solution

1. Data: Variable is systolic blood pressure, n=157 , sample mean=146, s=27, α=0.01.

2. Hypotheses: H_0 :μ=140; H_A: μ>140

3.Test Statistic:

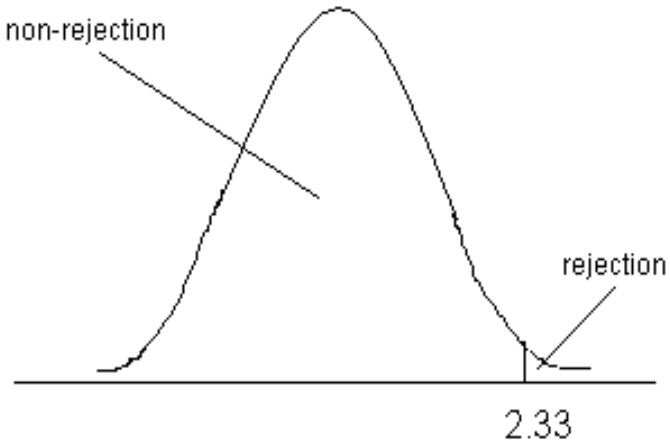
- $Z = \frac{\overline{X} - \mu_o}{\frac{s}{\sqrt{n}}} = \frac{146-140}{\frac{27}{\sqrt{157}}} = \frac{6}{2.1548} = 2.78$

4. Desicion Rule:

we reject H_0 if $Z>Z_{1-\alpha}$

$= Z_{0.99}= 2.33$

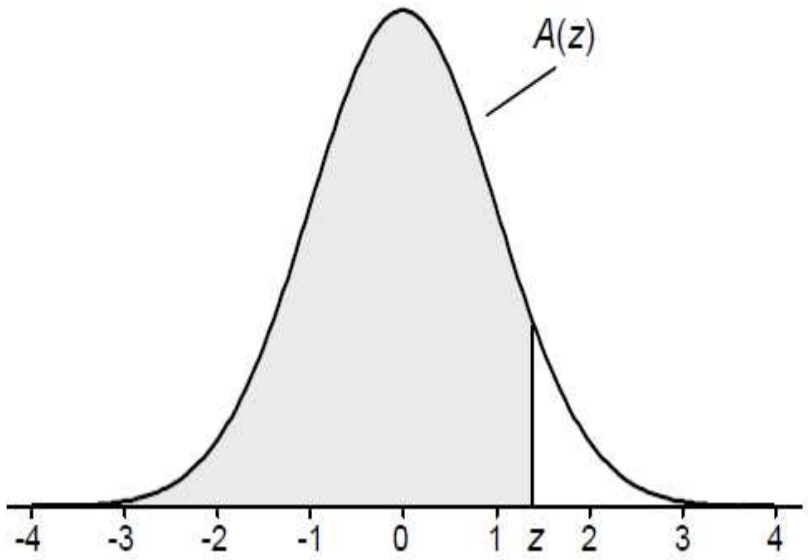
(from table A1)



5. Decision: Hence, we can conclude that the mean systolic blood pressure for a population of African-American is greater than 140 mm Hg.

TABLE A.1

Cumulative Standardized Normal Distribution



$A(z)$ is the integral of the standardized normal distribution from $-\infty$ to z (in other words, the area under the curve to the left of z). It gives the probability of a normal random variable not being more than z standard deviations above its mean. Values of z of particular importance:

z	$A(z)$	
1.645	0.9500	Lower limit of right 5% tail
1.960	0.9750	Lower limit of right 2.5% tail
2.326	0.9900	Lower limit of right 1% tail
2.576	0.9950	Lower limit of right 0.5% tail
3.090	0.9990	Lower limit of right 0.1% tail
3.291	0.9995	Lower limit of right 0.05% tail

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998

Student's *t-test*

- It is used to test the null hypothesis that there is no difference between the means of the two groups.
- There are three cases:

i) **one-sample *t-test*** *:* To test if a sample mean (as an estimate of a population mean) differs significantly from a given population mean.

The formula for one sample *t-test is =(x- u)/SE*

Where x = sample mean, u = population mean and SE = standard error of mean

ii) The unpaired t-test

To test if the population means estimated by two independent samples differ significantly.

The formula for unpaired t-test is: $t=(X_1-X_2)/SE$

where X_1-X_2 is the difference between the means of the two groups and SE denotes the standard error of the difference.

iii) The paired t-test

To test if the population means estimated by two dependent samples differ significantly. Usually, it is used when measurements are made on the same subjects before and after a treatment.

The formula for paired t-test is: d/SE

where d is the mean difference and SE denotes the standard error of this difference.

TABLE A.2

t Distribution: Critical Values of *t*

		Significance level					
Degrees of freedom	*Two-tailed test:* *One-tailed test:*	10% 5%	5% 2.5%	2% 1%	1% 0.5%	0.2% 0.1%	0.1% 0.05%
1		6.314	12.706	31.821	63.657	318.309	636.619
2		2.920	4.303	6.965	9.925	22.327	31.599
3		2.353	3.182	4.541	5.841	10.215	12.924
4		2.132	2.776	3.747	4.604	7.173	8.610
5		2.015	2.571	3.365	4.032	5.893	6.869
6		1.943	2.447	3.143	3.707	5.208	5.959
7		1.894	2.365	2.998	3.499	4.785	5.408
8		1.860	2.306	2.896	3.355	4.501	5.041
9		1.833	2.262	2.821	3.250	4.297	4.781
10		1.812	2.228	2.764	3.169	4.144	4.587
11		1.796	2.201	2.718	3.106	4.025	4.437
12		1.782	2.179	2.681	3.055	3.930	4.318
13		1.771	2.160	2.650	3.012	3.852	4.221
14		1.761	2.145	2.624	2.977	3.787	4.140
15		1.753	2.131	2.602	2.947	3.733	4.073
16		1.746	2.120	2.583	2.921	3.686	4.015
17		1.740	2.110	2.567	2.898	3.646	3.965
18		1.734	2.101	2.552	2.878	3.610	3.922
19		1.729	2.093	2.539	2.861	3.579	3.883
20		1.725	2.086	2.528	2.845	3.552	3.850
21		1.721	2.080	2.518	2.831	3.527	3.819
22		1.717	2.074	2.508	2.819	3.505	3.792
23		1.714	2.069	2.500	2.807	3.485	3.768
24		1.711	2.064	2.492	2.797	3.467	3.745
25		1.708	2.060	2.485	2.787	3.450	3.725

26	1.706	2.056	2.479	2.779	3.435	3.707
27	1.703	2.052	2.473	2.771	3.421	3.690
28	1.701	2.048	2.467	2.763	3.408	3.674
29	1.699	2.045	2.462	2.756	3.396	3.659
30	1.697	2.042	2.457	2.750	3.385	3.646
32	1.694	2.037	2.449	2.738	3.365	3.622
34	1.691	2.032	2.441	2.728	3.348	3.601
36	1.688	2.028	2.434	2.719	3.333	3.582
38	1.686	2.024	2.429	2.712	3.319	3.566
40	1.684	2.021	2.423	2.704	3.307	3.551
42	1.682	2.018	2.418	2.698	3.296	3.538
44	1.680	2.015	2.414	2.692	3.286	3.526
46	1.679	2.013	2.410	2.687	3.277	3.515
48	1.677	2.011	2.407	2.682	3.269	3.505
50	1.676	2.009	2.403	2.678	3.261	3.496
60	1.671	2.000	2.390	2.660	3.232	3.460
70	1.667	1.994	2.381	2.648	3.211	3.435
80	1.664	1.990	2.374	2.639	3.195	3.416
90	1.662	1.987	2.368	2.632	3.183	3.402
100	1.660	1.984	2.364	2.626	3.174	3.390
120	1.658	1.980	2.358	2.617	3.160	3.373
150	1.655	1.976	2.351	2.609	3.145	3.357
200	1.653	1.972	2.345	2.601	3.131	3.340
300	1.650	1.968	2.339	2.592	3.118	3.323
400	1.649	1.966	2.336	2.588	3.111	3.315
500	1.648	1.965	2.334	2.586	3.107	3.310
600	1.647	1.964	2.333	2.584	3.104	3.307
∞	1.645	1.960	2.326	2.576	3.090	3.291

Chi-square-test

It is used to analyze the categorical data.

It compares the frequencies and tests whether the observed data differ significantly from the expected data if there were no differences between groups (H0).

It is calculated by the sum of the squared difference between observed (O) and the expected (E) data divided by the expected (E) data.

$$\chi^2 = \sum_i \frac{(O_i - E_i)^2}{E_i}$$

The Decision Rule

- The quantity χ-square will be small if the observed and expected frequencies are close together and will be large if the differences are large.
- The computed value of χ-square is compared with the tabulated value with degrees of freedom = (r-1)(c-1) where r is the number of rows and c is the number of columns.
- Reject H_0, if χ-square is greater than or equal to the tabulated χ-square for the chosen value of α.

TABLE A.4

χ^2 (Chi-Squared) Distribution: Critical Values of χ^2

	Significance level		
Degrees of freedom	5%	1%	0.1%
1	3.841	6.635	10.828
2	5.991	9.210	13.816
3	7.815	11.345	16.266
4	9.488	13.277	18.467
5	11.070	15.086	20.515
6	12.592	16.812	22.458
7	14.067	18.475	24.322
8	15.507	20.090	26.124
9	16.919	21.666	27.877
10	18.307	23.209	29.588

References

- Biostatistics: **Basic Concepts and Methodology for the Health Sciences**, 10ed, ISV. Wayne W. Daniel, Chad L. Cross. ISBN: 9788126551897. 954 pages.
- Ali Z, Bhaskar SB. Basic statistical tools in research and data analysis. Indian J Anaesth. 2016 Sep;60(9):662-669. doi: 10.4103/0019-5049.190623. Erratum in: Indian J Anaesth. 2016 Oct;60(10):790. PMID: 27729694; PMCID: PMC5037948.

Thank you.

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