Lecture-VIII Programme: M. Sc./B.Sc. Physics

Wave Guide: Waveguide Resonators TE Mode

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- **Transverse Electric (TE) mode of rectangular resonator/ cavity**
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- **In this section we will discuss about Transverse Electric (TE) mode of rectangular wave guide resonator.**
- **Suppose a rectangular cavity (or closed conducting box) of dimensions along a, b, and c along the X-, Y- and Z- axes and it is represented in Figure 1.**
- **Here we will consider the positive *z*-direction as the "direction of wave propagation." In fact, there is no wave propagation.**

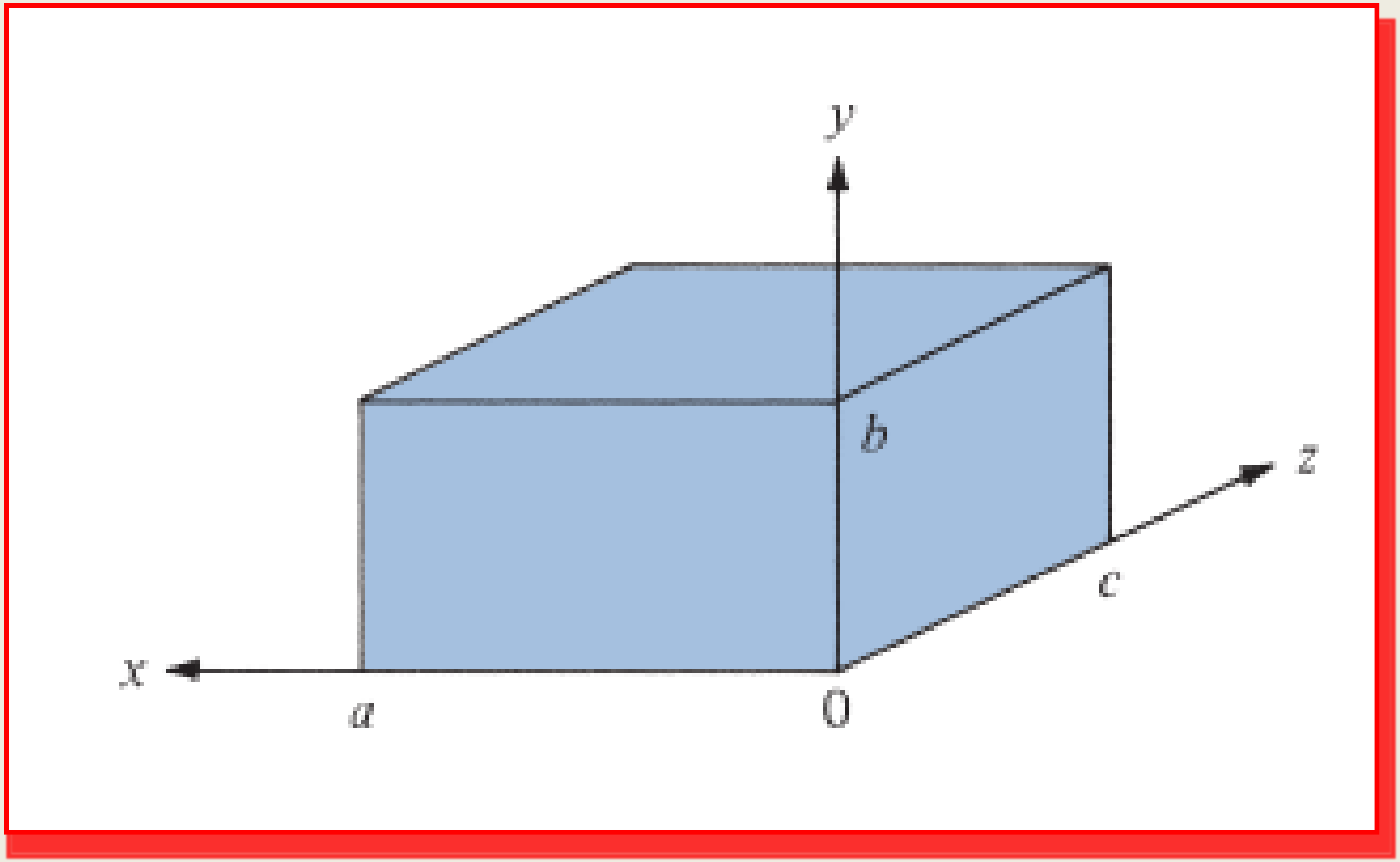

Figure 1: A rectangular resonator [*Ref-1] .

TE Mode to z in Rectangular resonator

❖ To find the expressions for the magnetic field for the propagation to z in TE mode, let us setting $E_z = 0$ and apply the method which we have used in previous lecture-IV for the formulation of TE modes in rectangular wave guide resonator. We get the following expression for the magnetic field-

$$H_{zs} = (b_1 \cos k_x x + b_2 \sin k_x x)(b_3 \cos k_y y + b_4 \sin k_y y)(b_5 \cos k_z z + b_6 \sin k_z z)$$

------------ [1]

❖ Now applying the following boundary conditions those are given as -

$$E_z = 0 \quad \text{at} \quad x = 0, a$$ ------------ [2]

$$E_z = 0 \quad \text{at} \quad y = 0, b$$ ------------ [3]

$$E_y = 0, E_x = 0 \quad \text{at} \quad z = 0, c$$ ------------ [4]

❖ As like shown in previous lecture-III, the conditions in equations (2 and 3) are satisfied, thus we get-

$$k_x = \frac{m\pi}{a}, \quad k_y = \frac{n\pi}{b}$$ ------------ [5]

❖ **Here we will consider the positive *z*-direction as the "direction of wave propagation." In fact, there is no wave propagation.**

❖**The boundary conditions in eq. (4) combined with equations (17-22 from previous lecture - II) gives-**

$$H_{zs} = 0 \quad \text{at} \quad z = 0, c \qquad \text{------------ [6]}$$

$$\frac{\partial H_{zs}}{\partial x} = 0 \quad \text{at} \quad x = 0, a \qquad \text{------------ [7]}$$

$$\frac{\partial H_{zs}}{\partial y} = 0 \quad \text{at} \quad y = 0, b \qquad \text{------------ [8]}$$

❖ **Imposing the conditions from equations (6-8) on equation (1) in the similar way as for TM mode to z leads to-**

$$H_{zs} = H_o \cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)\sin\left(\frac{p\pi z}{c}\right) \quad \text{-------------} \quad [9]$$

where *m* = 0, 1, 2, 3, . . . , *n* = 0, 1, 2, 3, . . . , and *p* = 1, 2, 3, ...

❖ **The phase constant β is determined as from previous lecture – VII. The required expression for the phase constant β is thus-**

$$\beta^2 = k^2 = \left[\frac{m\pi}{a}\right]^2 + \left[\frac{n\pi}{b}\right]^2 + \left[\frac{p\pi}{c}\right]^2$$

-------------- [10]

since

$$\beta^2 = \omega^2\mu\varepsilon$$

-------------- [11]

from eq. (10), we obtain the *resonant frequency* f_r

$$2\pi f_r = \omega_r = \frac{\beta}{\sqrt{\mu\varepsilon}} = \beta u'$$

$$f_r = \frac{u'}{2}\sqrt{\left[\frac{m}{a}\right]^2 + \left[\frac{n}{b}\right]^2 + \left[\frac{p}{c}\right]^2}$$

------------- [12]

- **This equation signifies that the resonant frequencies from a distance set mainly depending on the selection of *m*, *n* and *p*. since propagation in the cavity resonator take place in more than one direction and in several modes, cavity resonators in general have a great number of feasible modes of resonance.**
- **A cavity resonator for a precise application may be intended in such a way however, that only one mode of resonance is obtained over a restricted frequency range.**

- **From equation (17) it is clear that if any of two integers *m*, *n*, and *p* are zero, all the field components would turn into zero. This entails that TE_{000}, TE_{001}, TE_{010} , and TE_{100} , modes do not exist in the cavity resonator.**

- **The physical probable lowest modes are of the type TE_{101}, TE_{001} (where only one integer is zero).**

- **The smallest permitted frequency of these modes is known as the fundamental frequency of first harmonics.**

- **The higher permitted values which are integral multiples of the fundamental are known as *overtone* or *higher harmonics*.**
- **The mode that has the lowest resonant frequency for a given cavity size *(a, b, c)* is also called the *dominant mode*. If (*a* > *b* < *c*). Note that for (1/*a* < 1/*b* > 1/*c*), the resonant frequency of *TM_{110} mode is higher than that for TE_{101} mode*; hence, *TE_{101} is dominant*.**
- **When dissimilar modes have the similar resonant frequency, we say that the modes are *degenerate*; one mode will dominate others depending on how the cavity is excited.**

The Quality Factor of Rectangular Resonator

- **A realistic resonant cavity has walls with limited conductivity σ_c and is, thus, able of losing stored energy.**
- **The quality factor Q is a means of determining the loss.**

The quality factor is also a measure of the bandwidth of the cavity resonator.

It can be defined as

$$Q = 2\pi \cdot \frac{\text{time average energy stored}}{\text{energy loss per cycle of oscillation}} \quad \text{------------} \quad [13]$$

$$Q = 2\pi \cdot \frac{W}{P_L T} = \omega \frac{W}{P_L}$$ ------------- [14]

where $T = 1/f$ the period of oscillation, P_L is the time-average power loss in the cavity, and W is the total time-average energy stored in electric and magnetic fields in the cavity.

❖ In general Q is very high for a cavity resonator in comparison with Q value for an RLC resonant circuit.

❖ Mathematically the quality factor for the dominant TE_{101} is given by-

$$Q_{TE_{101}} = \frac{(a^2 + c^2)abc}{\delta[2b(a^3 + c^3) + ac(a^2 + c^2)]} \quad \text{------------- [15]}$$

where δ is the skin depth of the cavity walls.

which is defined as-

$$\delta = \frac{1}{\sqrt{\pi f_{101} \mu_o \sigma_c}} \quad \text{------------- [16]}$$

Numerical problems based on resonator cavity

1. **An air-filled resonant cavity with dimensions *a* = 5 cm, *b* = 4 cm, and *c* = 10 cm is made of copper $\sigma_c = 5.8 \times 10^7$ S/m. Find- (a) The five lowest-order modes (b) T he quality factor for TE_{101} mode.**
2. **For air filled, lossless cavity resonator of dimensions a = 60 cm, b = 50 cm and c = 40 cm, list, in order of ascending resonant frequencies, the ten modes.**
3. **Shielded rooms can be viewed as resonant cavities. Consequently, operation of equipment in such a room at a resonant frequency of the cavity should be avoided. A typical shielded room has size (408 inch) (348 inch) (142 inch). Determine its lowest resonant frequency.**
4. **Determine Q of the TE_{101} mode in an air –filled brass cavity having dimensions *a* = 4 cm, *b* = 4 cm, and *c* = 2 cm.**
5. **Design a cubic (a = b = c) cavity resonator to have a dominant resonant frequency of 7GHz. Also calculate Q of the dominant mode of the cavity. It is assuming that the walls are made with brass of conductivity $\sigma_c = 15.7 \times 10^6$ S/m.**
6. **A cubical cavity measures 3 cm on each side. What is the lowest resonant frequency?**

References:

1. *Elements of Electromagnetics, 2nd edition by M N O Sadiku.*

2. *Engineering Electromagnetics by W H Hayt and J A Buck.*

3. *Elements of Electromagnetic Theory & Electrodynamics, Satya Prakash*

- **For any query/ problem contact me on whatsapp group or mail on me**

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- **Next *** we will discuss the solutions of given problems of this lecture and more.**

Thank you