Hardy-Weinberg Equilibrium (HWE)
Exercise part-III
Core course: ZOOL 3014
B.Sc. (Hons'): VIth Semster
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1. **About 70 percent of all white North Americans can taste the chemical phenylthiocarbamide, and the remainder cannot. The ability to taste is determined by the dominant allele _T_, and the inability to taste is determined by the recessive allele _t_. If the population is assumed to be in Hardy-Weinberg equilibrium, what are the genotypic and allelic frequencies in this population?**

Because 70 percent are tasters (_T_/_T_), 30 percent must be nontasters (_t_/_t_). This homozygous recessive frequency is equal to q^2; so, to obtain q, we simply take the square root of 0.30:

$$q = \sqrt{0.30} = 0.55$$

Because $p + q = 1$, we can write

$$p = 1 - q = 1 - 0.55 = 0.45$$

Now we can calculate

$$p^2 = 0.45^2 = 0.20(T/T)$$
$$2pq = 0 \times 0.45 \times 0.55 = 0.50(T/t)$$
$$q^2 = 0.3(t/t)$$

2. **In a large natural population of _Mimulus guttatus,_ one leaf was sampled from each of a large number of plants. The leaves were crushed and subjected to gel electrophoresis. The gel was then stained for a specific enzyme X. Six different banding patterns were observed, as shown in the diagram.**

a.Assuming that these patterns are produced by a single locus, propose a genetic explanation for the six types.
b.How can you test your idea?
c.What are the allelic frequencies in this population?
d.Is the population in Hardy-Weinberg equilibrium?
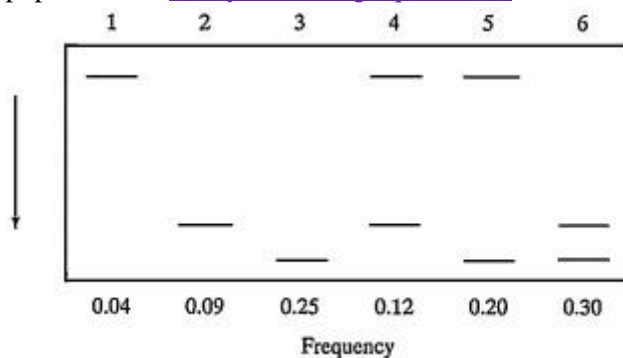
a. Inspection of the gel reveals that there are only three band positions: we will call them slow, intermediate, and fast. Furthermore, any individual can show either one band or two. The simplest explanation is that there are three al-leles of one locus (let's call them _A_ S, _A_ I, and _A_ F) and that the individuals with two bands are heterozygotes. Hence, 1 = S/S, 2 = I/I, 3 = F/F, 4 = S/I, 5 = S/F, and 6 = I/F.

b. The hypothesis can be tested by making controlled crosses. For example, from a self of type 5, we can predict 1/4 S/S, 1/2S/F, and 1/4 F/F.

c. The frequencies can be calculated by a simple extension of the two-allele formulas. Hence:

$$f(S) = 0.04 + \tfrac{1}{2}(0.12) + \tfrac{1}{2}(0.20) = 0.20$$
$$f(I) = 0.09 + \tfrac{1}{2}(0.12) + \tfrac{1}{2}(0.30) = 0.30$$
$$f(F) = 0.25 + \tfrac{1}{2}(0.20) + \tfrac{1}{2}(0.30) = 0.50$$

d. The Hardy-Weinberg genotypic frequencies are:

$$(p + q + r)^2 = p^2 + q^2 + r^2$$
$$+ 2pq + 2pr + 2qr$$

$$= 0.04 + 0.09 + 0.25 + 0.12$$
$$+ 0.20 + 0.30$$

which are precisely the observed frequencies. So it appears that the population is in equilibrium.

3. In a large experimental _Drosophila_ population, the <u>fitness</u> of a <u>recessive phenotype</u> is calculated to be 0.90, and the <u>mutation rate</u> to the <u>recessive allele</u> is 5×10^{-5}. If the population is allowed to come to equilibrium, what allelic frequencies can be predicted?

Here <u>mutation</u> and selection are working in opposite directions, so an equilibrium is predicted. Such an equilibrium is described by the formula

$$\hat{q} = \sqrt{\frac{\mu}{s}}$$

In the present question, $\mu = 5 \times 10^{-5}$ and $s = 1 - W = 1 - 0.9 = 0.1$. Hence

$$\hat{q} = \sqrt{\frac{5 \times 10^{-5}}{10^{-1}}} = 2.2 \times 10^{-2} = 0.022$$

$$\hat{p} = 1 - 0.022 = 0.978$$

4. Suppose 25 out of 750 students are redheads. What is frequency of redheads? If a random student is choosen, what is the probability they are a redhead?

- Freq (Redheads) = 25/750 = 0.033 or 3.3 percent
- Probability of a Redhead = 25/750, or 3.3 percent

5. Consider a locus with two alleles, A and a. If the frequency of AA is 0.25, what is the frequency of A under Hardy-Weinberg?

- Under H-W, if $p = \text{freq}(\mathbf{A})$, then $p^2 = \text{freq}(\mathbf{AA})$, hence $p^2 = 0.25$ or $p = 0.5$.

6. If the genotypes AA, Aa, and aa have frequencies 0.5, 0.25, and 0.25 (respectively), what are p = freq(A)? q = freq (a)? After a single generation of random mating, what is the expected frequency of AA? of Aa? of aa?

- $p = \text{freq}(\mathbf{AA}) + (1/2) \text{freq}(\mathbf{Aa}) = 0.5 + (1/2)(0.25) = 0.625$.

- $q = 1\text{-}p = 0.375$

- $\text{freq}(\mathbf{AA}) = p^2 = 0.625^2 = 0.391$

- $\text{freq}(\mathbf{Aa}) = 2pq = 2*0.625*0.375 = 0.469$

- $\text{freq}(\mathbf{aa}) = q^2 = 0.375^2 = 0.140$

7. Consider a locus with 12 alleles, A_1, A_2, ... , A_{12}. What is the frequency of allele A_1 if we know that freq(A_1A_1) = 0.10, and that the frequency of all heterozygote genotypes containing A_1 is 0.40. Under the assumption of Hardy-Weinberg, what is the expected frequency of A_1A_1? Of any heterozygote involving A_1?

- $\text{freq}(\mathbf{A_1}) = \text{freq}(\mathbf{A_1A_1}) + (1/2)(\text{all } \mathbf{A_1} \text{ heterozygotes}) = 0.10 + (1/2)(0.4) = 0.3$
- $\text{freq}(\mathbf{A_1A_1}) = \text{freq}(\mathbf{A_1})^2 = 0.3^2 = 0.09$
- $\text{freq}(\mathbf{A_1A_x}) = 2*\text{freq}(\mathbf{A_1})(1\text{- } \text{freq}(\mathbf{A_1})) = 2*0.3*0.7 = 0.42$

8. Assume Hardy-Weinberg and a locus with two alleles. If p = freq(A), what fraction of all copies of allele a. are found in heterozygotes?

- $\text{freq}(\mathbf{a}) = p = (1/2)\text{freq}(\mathbf{Aa}) + \text{freq}(\mathbf{aa})$, so that the fraction of $\mathbf{a}$ found in heterozygotes is $(1/2)\text{freq}(\mathbf{Aa}) / \text{freq}(\mathbf{a}) = (1/2) \, 2p(1\text{-}p) / (1\text{-}p) = p$

9. Assume that A is completely dominant over a, with aa individuals displaying a disease while all others are normal. An individual of genotype is Aa is called a carrier as they appear normal, but can have offspring with the disease.

(i) Under Hardy-Weinberg, if freq(**a**) = 0.001 what fraction of all copies of **a** are in carriers?

o From Problem 5, fraction in carriers= freq(A) = 1-.001 =0.999

(ii) What fraction of all **aa** individuals had both parents as carriers (i.e., neither parent displays the trait).

o The frequency of **Aa** x **Aa** matings is (because of random mating) (2pq)*(2pq), with (1/4) of the offspring being **aa**, hence since the frequency of **aa** is q^2, the fraction of these offpsring having only carriers as parents is $(1/4)*(2pq)*(2pq) / q^2 = p^2 = 0.999^2 = .998$

10. If the genotypes AA, Aa, and aa have fitnesses 1: 1.5: 1.6, what allele is fixed?

o Allele **a**, as genotype **aa** has the highest fitness.

11. The genotype aa is lethal and yet the population has an equilibrium frequency for a of .40. If the fitness of Aa is 1, what is the fitness of the AA genotype?

o Recall if the genotypes **AA**: **Aa**: **aa** have fitnesses 1-s : 1 : 1-t, then the equilibrium frequency of **A** is t/(s+t).
  ▪ Here, t = 1 (as aa is lethal), so that freq(**A**) = 1- freq(**a**) =0.6 = 1/(1+s)
  ▪ Solving gives 1+s = 1/0.6, or s = 1/0.6 -1 = 2/3
  ▪ Hence fitness of **AA** = 1-s = 1-2/3 = 1/3

12. Suppose a population starts out with 10,000 AA individuals, 20,000 Aa, and 10,000 aa.

(i) What is the frequency of **A**? Is this population in Hardy-Weinberg?

- o freq(**AA**)=freq(**aa**) = 1/4, freq(**Aa**) =1/2.
- o freq(**A**) =freq(**AA**)+(1/2)freq(**Aa**) = 1/4 + (1/2)*(1/2) = 1/2
- o Under HW expect freq(**AA**) = p^2 =1/4, freq(**Aa**) = 2pq =1/2, and freq(**aa**) = q^2 =1/4 which is indeed what we see.

(ii) Suppose all **aa** individuals die before reproducing, while (on average) **AA** and **Aa** individuals leave the same number of offspring. What are the fitnesses of these three genotypes?

- o fitness of **aa** = 0, while fitness of **AA** and **Aa** are the same, and we set this equal to one.

(iii) Following selection, what fraction of the surviving adults are **AA**? **Aa**? **aa**? What is the frequency of **a**?

- o After selection, freq(**aa**) = 0, while the proportion of of **AA** and **Aa** is 1:2 after selection, so that freq(**AA**) = 1/(1+2) = 1/3 and freq(**Aa**) = 2/(1+2) = 2/3. Hence, freq(**a**) after selection = freq(**aa**) + (1/2)freq(**Aa**) = 0 + (1/2)(2/3) = 1/3.

(iv) If these surviving adults mate at random, what is the frequency of **a** in the next generation (before selection acts).

- o Here freq(**a**) = 1/3, freq(**A**) = 2/3, giving
  - ▪ freq(**AA**) = $(2/3)^2$ = 4/9,

- freq(**aa**) = $(1/3)^2$ = 1/9,
- freq(**Aa**) = 2*(1/3)(2/3)= 4/9.

13. In a certain population of newts, being poisonous (P) is dominant over not being poisonous (p). You count 200 newts, and 8 are not poisonous. What are the allele frequencies of the parent population?

1. $p^2 + 2pq + q^2 = 1$

2. PP   Pp   pp

3. Poisonous $(p^2 + 2pq)$ not poisonous (q^2)

4 Poisonous (192) not poisonous (8)

So you know that q^2 = 8/200 = 0.04.

To find q, simply take the square root of q^2 : q = $\sqrt{.04}$ = 0.2 p = 1-q = 1 – 0.2 = 0.8

14. Fifty newts are washed downstream after a big storm and colonize a new pond. What do you expect the frequency and number of each genotype to be?

You would expect the allele frequencies to remain the same,

so q = 0.2 and p = 0.8.

To find the genotype frequencies, fill in the Hardy-Weinberg equation.

$p^2 = (0.8)^2 = 0.64$

$2pq = 2*0.2*0.8 = 0.32$

$q^2 = (0.2)^2 = 0.04$

To find the number of each genotype, multiply the total population by the genotype frequency.

PP $(p^2$) = 50*.64 = 32

Pp (2pq) = 50*0.32 = 16

pp $(q^2$) = 50*0.04 = 2

15. Cystic fibrosis is a genetic disorder in homozygous recessives that causes death during the teenage years. If 4 in 10,000 newborn babies have the disease, what are the expected frequencies of the three genotypes in newborns, assuming the population is at HardyWeinberg equilibrium? Why is this assumption not strictly correct?

In a population of 10,000 newborn babies, the expected proportion of babies having the disease is 4/10,000 = 0.0004, which is q^2

The frequency of the allele q that causes the disease if the square root of this number,

$\sqrt{0.0004} = 0.02$.

You know that p + q = 1, therefore p = 1 – 0.02 = 0.98.

Now, you can calculate the expected frequency of the three genotypes:

p^2 (homozygote, normal) = 0.9604

2pq (heterozygote, normal) = 0.0392

q^2 (recessive, cystic fibrosis) = 0.0004

Assuming Hardy-Weinberg in this case is not correct because there is natural selection acting against the allele that causes the disease, given that children with the disease die before they reproduce (Remember that Natural Selection is defined as differential reproduction and/or survival among the individuals in a population).

16. The allele y occurs with a frequency of 0.8 in a population of clams. Give the frequency of genotypes YY, Yy, and yy.

The allele y has a frequency q = 0.8.

You know that p + q = 1,

then p = 1 – 0.8 = 0.2.

Now you can estimate the frequency of each genotype:

YY genotype frequency = p^2 = 0.04

Yy genotype frequency = 2pq = 0.32

yy genotype frequency = q^2 = 0.64.