

MAHATMA GANDHI CENTRAL UNIVERSITY

ECON3020:Theory of
Economic Growth

- Course Code:
ECON3020R

Solow model

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Solow model

- **The Solow model is design to show how growth is the capital stock, growth is lob our force and advances in technology interact is a economy and how they effect total output of a country**

Assumption of Solow

- i. The two factors of production, labour and capital.
- ii. There are constant returns to scale.
- iii. Labour and capital are substitutable to each other.
- iv. Marginal productivity of capital is diminishing
- v. There is capital saving, technical progress with growth rate “ t ”.
- vi. The saving ratio is constant ‘ s ’.
- vii. Growth rate of population is constant ‘ n ’.
- viii. Rate of depreciation is constant.
- ix. Economy is closed.
- x. Role of government is not considered.

Saving Population growth & Technology Progress

- Initially we assume that population is constant and technological Progress is not take place. We only study the impact of saving on capital stock .

Supply and demand for goods

- Just like in any economic model the equilibrium in Solow model is obtained through the interaction of supply and demand. Supply determined how much output is being produced, while demand determined how it is allocated among various factor.

The supply of goods & Production function

- The supply of goods in the Solow model is based upon the production function which relates outputs with factor input. Solow model assumes that there are only two factor inputs, capital and labour.

$$Y=F(L,K)$$

Where Y =GDP, K =capital, L =Labour Factor

Solow model assumes that production function exhibits the constant return to scale.

$$Y/L=F(L/L, K/L)$$

$$Y/L=F(1, K/L)$$

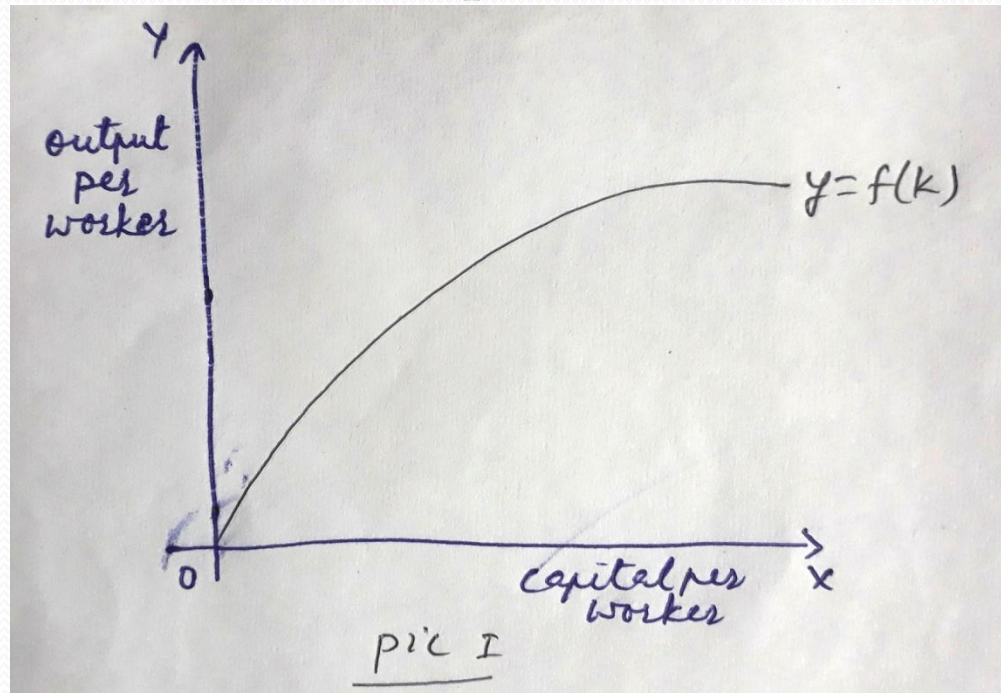
Let $Y/L=y$ (output per worker), $K/L=k$ (capital per worker)

$$y=F(1,k)$$

$$y=f(k)$$

Slope of production function = $\partial f/\partial k$ or Marginal product of capital

- The Solow model assumes that the marginal product of capital is diminishing which means as we increase the capital stock the output increases but at decreasing rate. This gave Concave slope



Demand for goods & the consumption function

- The Solow model assumes that the demand for goods comes from consumption and investment

$$Y=C+I$$

Divide both side by L

$$Y/L=C/L+I/L$$

➤ $Y=c+i$

C=consumption per worker

i=investment per worker

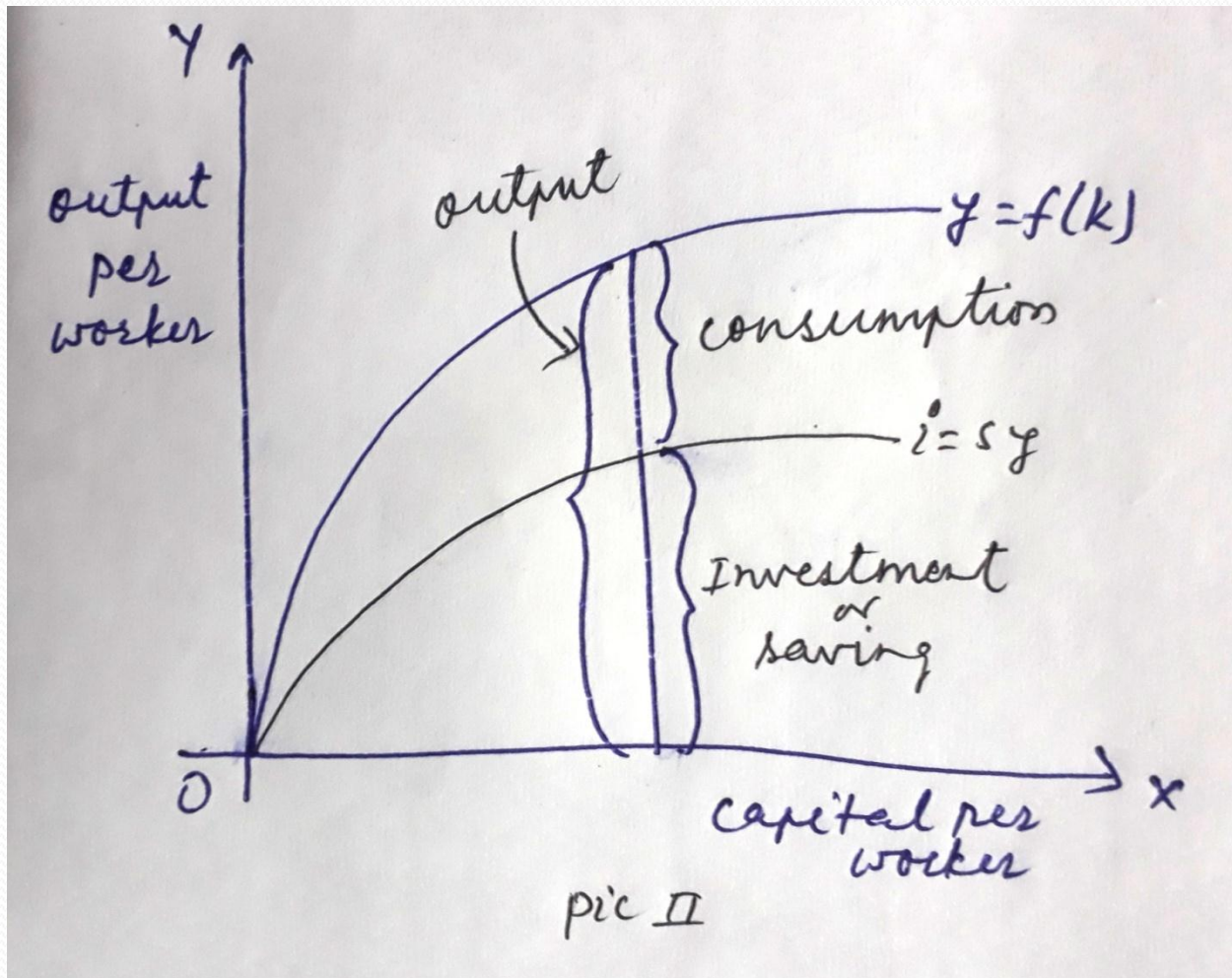
The solow model has omitted the government Purchases and net export .

Solow model assumes that every year People save a fixed Proportion of their income .

Couti....

- s =saving rate $0 \leq s \leq 1$
 $s = S/Y = S/L/Y/L$
- $1-s$ =consumption rate
- $1-s = c/y$ or C/Y
- $C = (1-s)y$
- $y = c + i$
- $Y = (1-s)y + i$
- $Y - y + sy = I$
- $Sy = I$
- This equation tells the investment equal saving .

Conti...



Growth in the capital stock

- There are two factor which affect the capital stock of an economy
 - i. Investment
 - ii. Depreciation

Investment is the accumulation of new capital stock. It refers to Purchase of plant & machinery tools , equipment etc. It causes capital stock to rise .

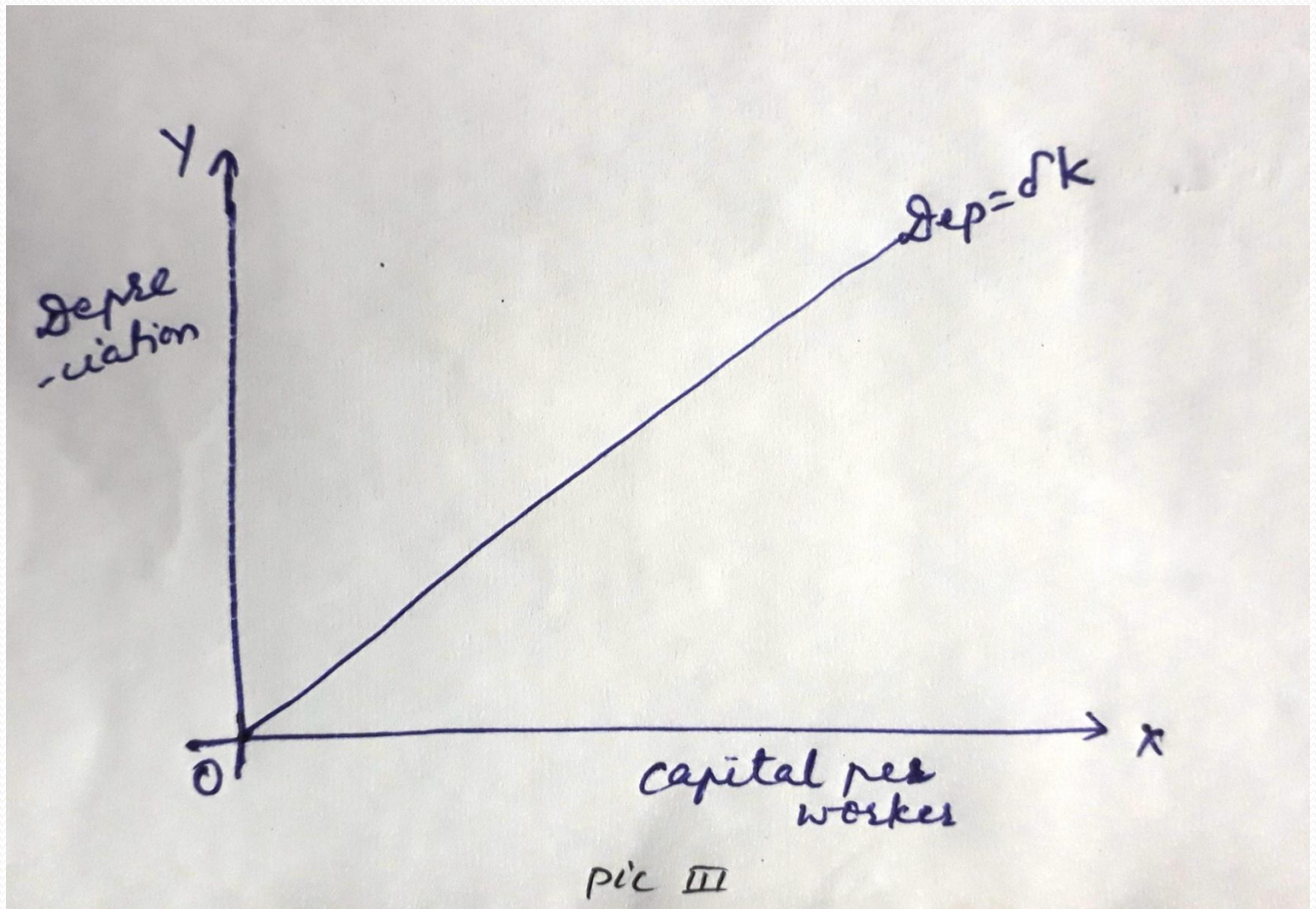
In solow model, it is assumed that investment is equal to saving. i,e $i=sy$

Depreciation refers to fall in the value of capital stock due to wear and tear.

Solow model assumes that capital stock depreciate at a constant rate δ

Depreciation= δk

Conti....



Conti.....

On the basis of there two factor we can conclude that

Change in Capital = Investment - Depreciation
stock

- $\Delta k = i - \delta k$
- $\Delta k = sf(k) - \delta k$
- $0 = sf(k) - \delta k$
- $k/f(k) = s / \delta$ - it will give the value of Capital

Steady state level of Capital

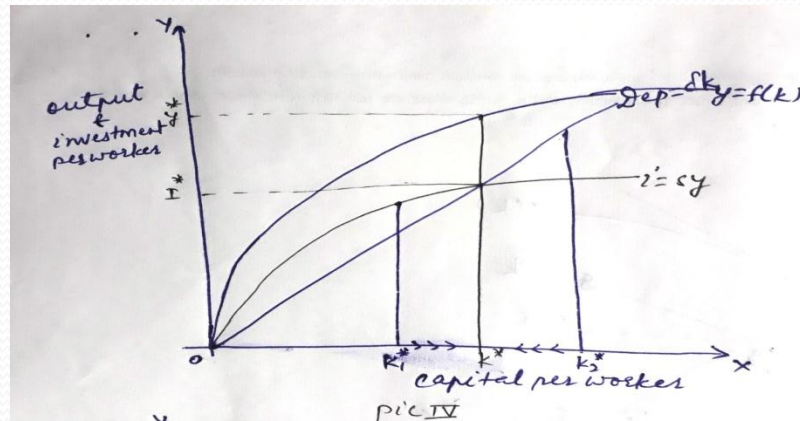
Steady state level of capital is such level of capital at which the capital stock not changing, i.e

$\Delta k=0$, which Means

➤ $i-\delta k=0$

➤ $i= \delta k$

So the level of capital for which $i= \delta k$ is called steady state level of capital $k^*=i= \delta k$ → steady state level of capital is represented as k^* .



Conti....

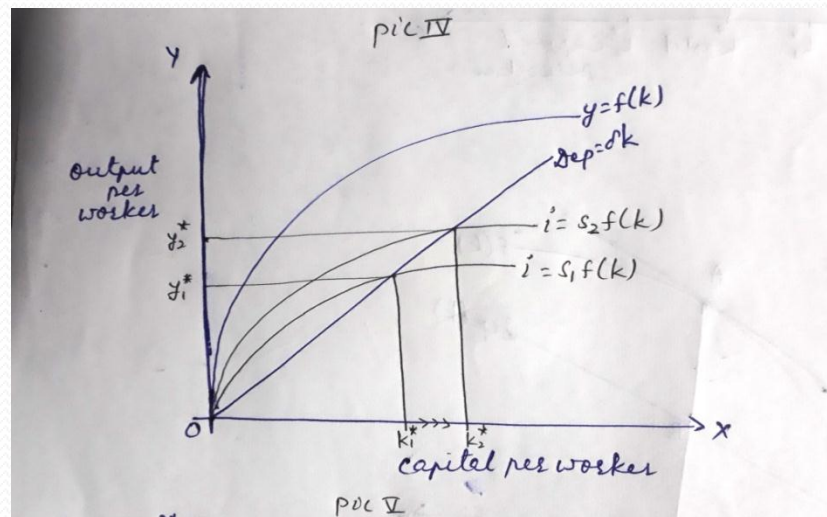
Steady state level of capital is known as long such equilibrium of the economy and it is because of the following reasons :-

- i. If the level of capital is not as steady state then it keeps an approaching towards the steady state.
- ii. If capital stock is at steady state level then it will stay their for even unless, there is any change in saving rate or the depreciation rate.

Effect of change in saving rate

Let us consider a situation in which saving rate of a economy increases, we assume that before increase the saving rate was s_1 and at the saving rate the economy was steady state level of capital k^*

Initial saving rate = s_1 Initial steady state of capital = k_1^* The saving rate rises from s_1 to s_2 this cause upward shift of investment schedule.



Conti....

At inhale steady state k_1^* the investment become more than the depreciation this causes increases in the capital stock. This process continue until economy reaches at new steady state level at k_2^*

At k_2^* both the steady state level of capital and steady level of output is higher than k_1^* , thus we can conclude that if the saving rate is high, the economy will have large capital stock and high level of output at the steady state and vice versa .

Growth effect vs level effect

Any change in a variable which causes the output to keep uprising contiguously is called growth effect Any change in a variable which increases the output from one level to another level is called level effect.

Any change in saving rate produces a level effect . Higher saving leads to faster growth in the Solow model but only temporarily . An increase in the saving rate, increases growth until the economy reaches a new state. So output increases only one level from another level with higher saving .

Golden rule level of capital

A benevolent policy maker would what to choose a steady state with the highest level of consumption. The steady state value of capital which maximizes consumption is called “Golden rule level of capital”. It is represented as “ k^* ” Gold.

Derivation of golden rule level of capital

since,

$$y=c+i$$

➤ $c=y-i$

➤ $c=f(k)-i$

For steady state , i =Depreciation

➤ $i=\delta k$

$$c^*=f(k^*)-\delta k$$

Differentiate with respect to k^*

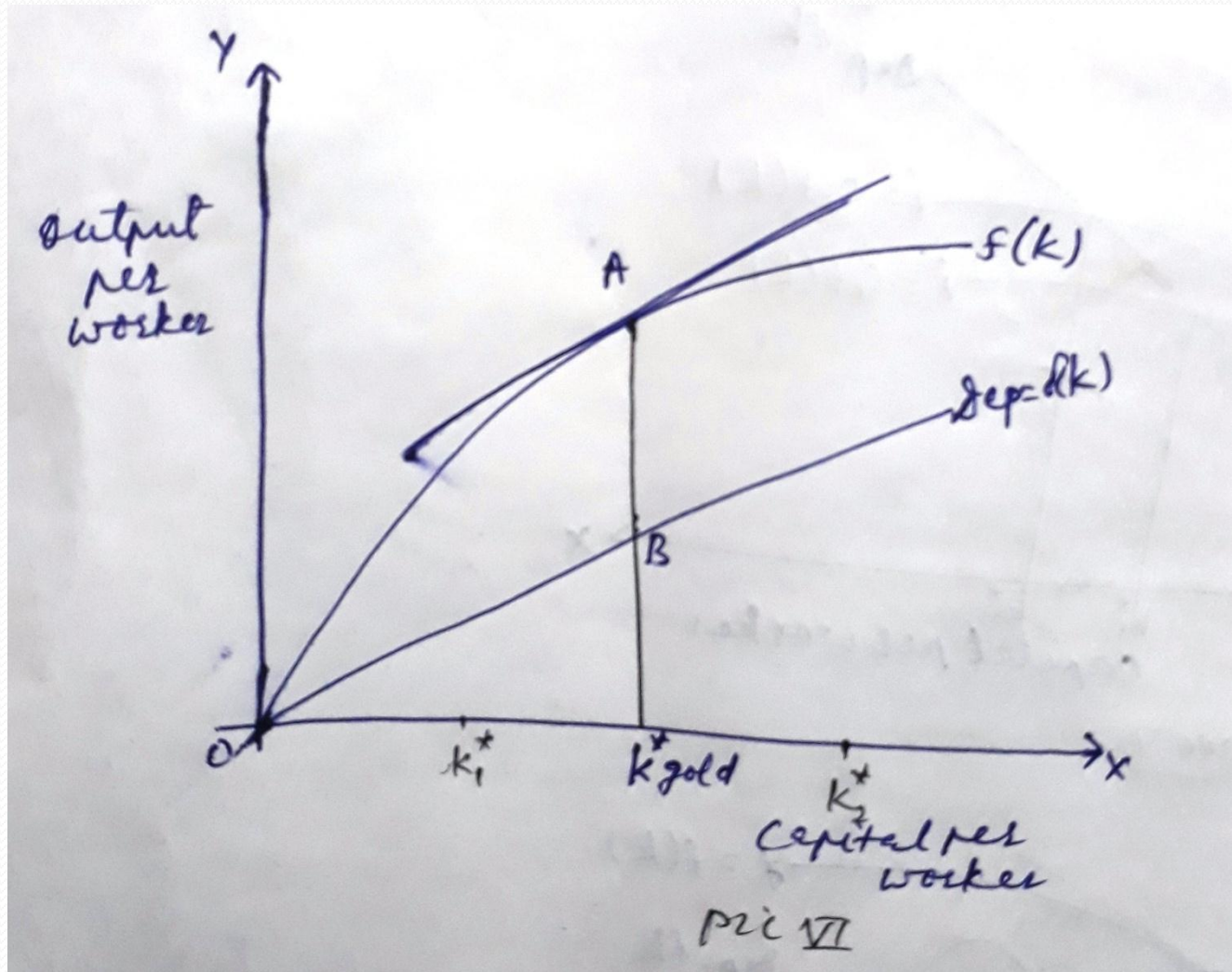
$$\partial c^*/\partial k^*=MPK-\delta$$

From 1st order condition

$$\partial c^*/\partial k^*=0 \longrightarrow MPK=\delta$$

$$\partial^2 k^*/\partial k^{*2}=\partial/\partial k^* (MPK) < 0$$

Conti....



Conti...

For golden rule level, it is assumed that the saving rate is being set by the policy maker and he fixed such saving rate at which the steady state consumption is maximum.

Economy automatically reaches at steady state but a deliberate action is required from policy maker to take it towards golden rule level. There are large number of steady state which economy can acquire but every economy has a unique golden rule level of capital.

If capital stock is below the golden rule level say at k_1 , then any increase in capital stock increases the output more than depreciation which increases consumption.

however if capital stock is above the golden rule level, an increase in capital stock increases output less than increase in depreciation. So consumption falls.



Thank you