Lecture-XII Programme: M. Sc. Physics

RADIATION: ELECTRIC DIPOLE RADIATION

Dr. Arvind Kumar Sharma (Assistant Professor)

Department of Physics, Mahatma Gandhi Central University, Motihari: 845401, Bihar

Contents

- **A stationary charge produces *electrostatic field* only. A uniformly moving charge generates *electromagnetic field* of short range, so they can not radiate *electromagnetic energy*. Thus the EM energy can radiate only if a charged particle is *accelerated*.**
- ***Radiation* may be thought of as the process of transmitting electric energy.**

Radiation

????

When charges *accelerate*, their fields can transport energy irreversibly out to infinity-a process we call *radiation*.

- The fields created by moving charges can be obtained either by the idea of retarded potentials or the theory of relativity.
- In the former case the field point parameters are connected to the source point. Since the cause happens at the source point and the effect follows at the field point. Consequently the process to relate the cause and the effect is known as ***retardation***.

- Let us suppose the source is localized close to the origin; we would like to determine the energy that is radiating at moment t_0. Envisage a huge sphere, out at radius r (Fig. 1).

- The power crossing through its surface is defined by the integral of the Poynting vector: .

$$P(r,t) = \oint \mathbf{S} \cdot d\mathbf{a} = \frac{1}{\mu_0} \oint (\mathbf{E} \times \mathbf{B}) \cdot d\mathbf{a}. \quad \text{-------------- [1]}$$

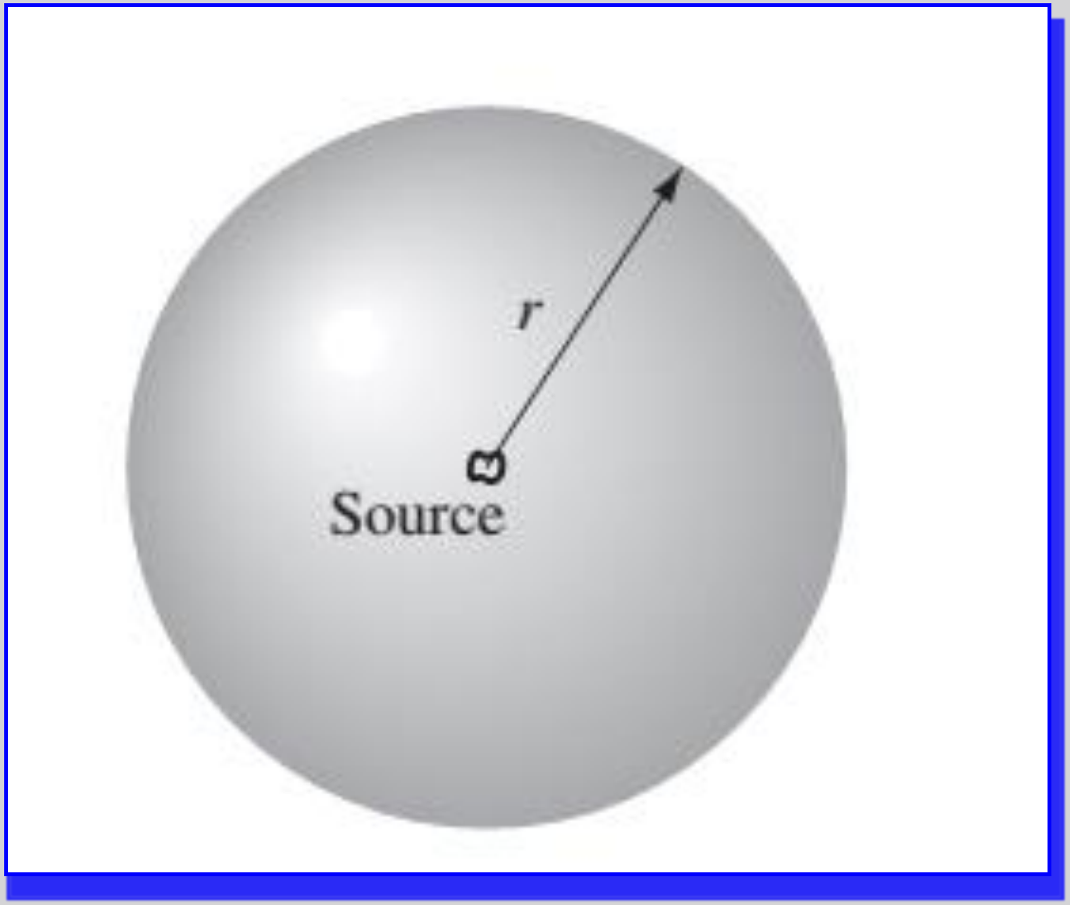


FIGURE 1: Representation of radiated power through a huge sphere of radius r. **[REF-1]

❖ **Since electromagnetic "information" moves with speed of light, this energy in fact left the source at the earlier time $t_0 = t - r/c$, therefore the power radiated is-**

$$P_{\mathrm{rad}}(t_0) = \lim_{r \to \infty} P\left(r, t_0 + \frac{r}{c}\right) \qquad \text{------------- [2]}$$

(with t_0 held constant). This is energy (per unit time) that is transmitted away and never comes back.

❖ Now, the area of the sphere is $4\pi r^2$, so for radiation to take place the Poynting vector must reduce (at large r) no faster than $1/r^2$ (if it went like $1/r^3$, for instance, then P would go like 1/r, and radiated power P_{rad} would be zero).

❖ Now from Coulomb's law, electrostatic fields drop off like $1/r^2$ (or even faster, if the total charge is nil), and the Biot-Savart law states that magnetostatic fields fall off like $1/r^2$ even quicker), which means that Poynting vector $S \sim 1/r^4$, for static arrangements. subsequently static sources do not radiate.

- However Jefimenko's (Equations 16 and 21 from previous lecture-IX) designate that time-dependent fields consist of terms (linking $\dot{\rho}$ and $\dot{\mathbf{J}}$) that fall off like 1/r; these are the terms that are liable for electromagnetic radiation.
- The study of radiation, then, engages option out the parts of E and B that go like 1/r at large distances from the source, making from them the $1/r^2$ term in S, integrating over a big spherical surface, and putting the limit as $r \rightarrow \infty$.

Electric Dipole Radiation

❖ **Figure 2 is showing two tiny metal spheres separated by a distance *d* and linked by a fine wire ; at instance *t* the charge on the upper sphere is defined by *$q(t)$*, and the charge on the lower sphere is *$-q(t)$*.**

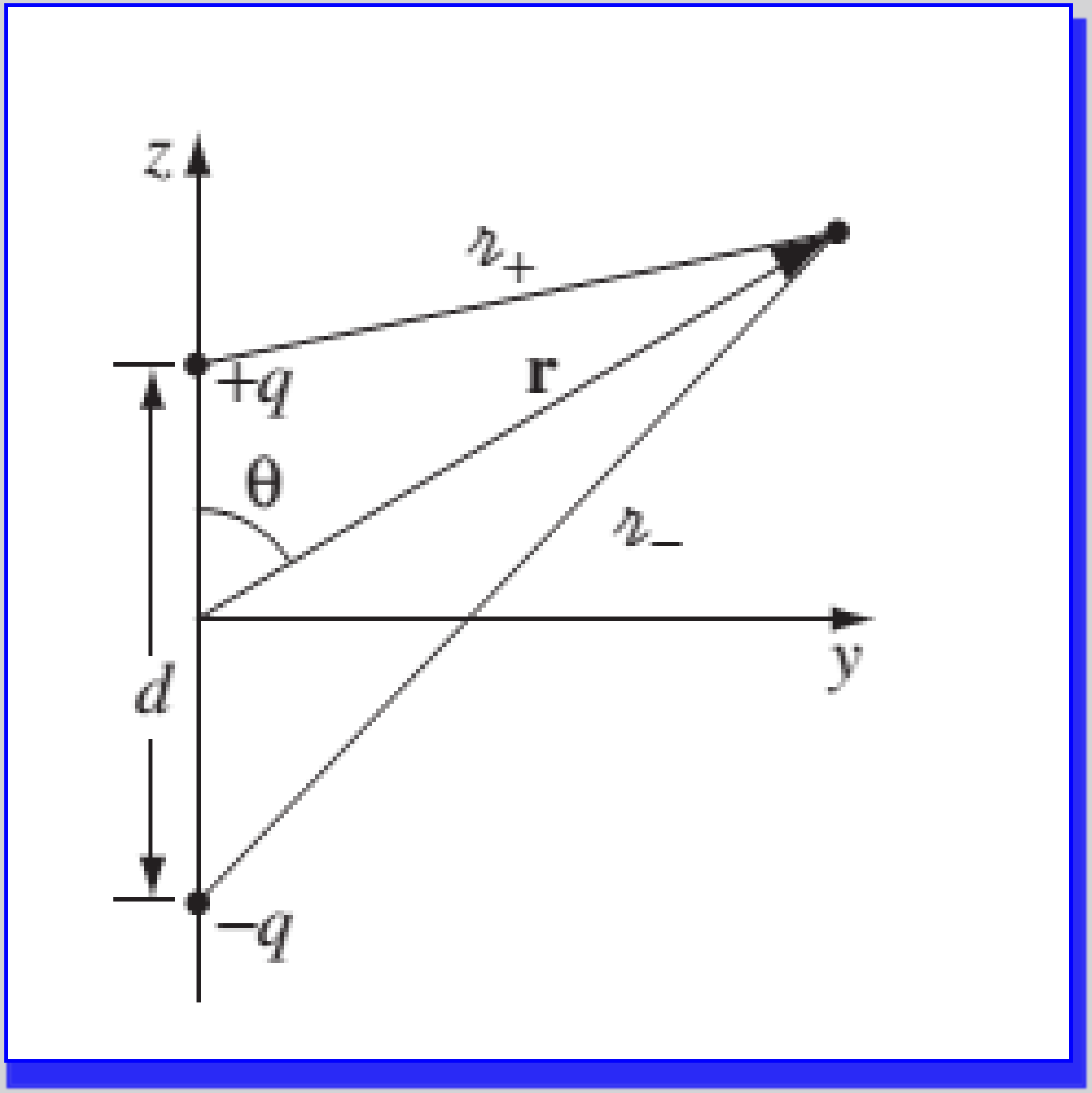


FIGURE 2: Representation of Electric dipole separated by a small distance d **[REF-1]

❖ **Considering that we drive the charge back and forth through the wire, from one end to the other, by an angular frequency *ω,* and the expression for this varying charge is defined as*:***

$$\boxed{q(t) = q_0 \cos(\omega t).}$$ ------------- [3]

The result is an oscillating electric dipole

$$\boxed{\mathbf{p}(t) = p_0 \cos(\omega t)\,\hat{\mathbf{z}},}$$ ------------- [4]

Where $\boxed{p_0 \equiv q_0 d}$ **is the maximum value of the dipole moment.**

❖ As we know that from previous lecture the retarded potential is

$$V(\mathbf{r}, t) = \frac{1}{4\pi\epsilon_0}\left\{\frac{q_0\cos[\omega(t - \mathscr{r}_+/c)]}{\mathscr{r}_+} - \frac{q_0\cos[\omega(t - \mathscr{r}_-/c)]}{\mathscr{r}_-}\right\} \quad \text{-------} \ [5]$$

where, from cosines formula,

$$\mathscr{r}_\pm = \sqrt{r^2 \mp rd\cos\theta + (d/2)^2}$$

❖ Now, to build this *physical* dipole into a *perfect* dipole, for this we consider that separation distance to be extremely small:

Approximation 1 : *d<< r*

❖ Obviously, if *d* is *zero* we find no potential at all; what we want is an expansion carried to *first order in d.* Thus-

$$\boxed{\mathscr{r}_\pm \cong r\left(1 \mp \frac{d}{2r}\cos\theta\right)}$$

We can write as-

$$\boxed{\frac{1}{\mathscr{r}_\pm} \cong \frac{1}{r}\left(1 \pm \frac{d}{2r}\cos\theta\right),} \qquad \text{-------------- [6]}$$

and

$$\boxed{\begin{aligned}\cos[\omega(t-\mathscr{r}_\pm/c)] &\cong \cos\left[\omega(t-r/c) \pm \frac{\omega d}{2c}\cos\theta\right] \\ &= \cos[\omega(t-r/c)]\cos\left(\frac{\omega d}{2c}\cos\theta\right) \\ &\quad \mp \sin[\omega(t-r/c)]\sin\left(\frac{\omega d}{2c}\cos\theta\right)\end{aligned}} \qquad \text{-------------- [7]}$$

❖ In the perfect dipole limit we have, further,

Approximation 2: $$\boxed{d \ll \frac{c}{\omega}}$$

(As waves of frequency *ω* have a wavelength *λ* = 2*πc*/*ω*, this amounts to the necessity *d* << *λ*.) Beneath these circumstances ,

$$\boxed{\cos[\omega(t - r_{\pm}/c)] \cong \cos[\omega(t - r/c)] \mp \frac{\omega d}{2c} \cos\theta \sin[\omega(t - r/c)].}$$

------------- [8]

❖ **Substituting eqs. 6 and 8 into eq. 5, we find the potential of an oscillating perfect dipole:**

$$V(r,\theta,t) = \frac{p_0 \cos\theta}{4\pi\epsilon_0 r}\left\{-\frac{\omega}{c}\sin[\omega(t-r/c)] + \frac{1}{r}\cos[\omega(t-r/c)]\right\}$$

-------------- [9]

❖ **In the static limit where (ω → 0) the second term reproduces the old relation for the** ***potential of a stationary dipole*** **which is given as:**

$$V = \frac{p_0 \cos\theta}{4\pi\epsilon_0 r^2}$$

-------------- [10]

This is not, though, the term that concerns us now; we are concerned in the fields that survive at *large distances from the source*, in the so-called radiation zone:

Approximation 3 : ***r >> c/ω or r >> λ***

λ is **the wavelength. In this region the potential reduces to**

$$\boxed{V(r,\theta,t) = -\frac{p_0\omega}{4\pi\epsilon_0 c}\left(\frac{\cos\theta}{r}\right)\sin[\omega(t - r/c)].}$$

------------- [11]

❖ **Now, the *vector* potential is determined by the current flowing in the wire:**

$$\mathbf{I}(t) = \frac{dq}{dt}\,\hat{\mathbf{z}} = -q_0\omega \sin(\omega t)\,\hat{\mathbf{z}}. \qquad \text{-------------- [12]}$$

From Fig. 3,

$$\mathbf{A}(\mathbf{r}, t) = \frac{\mu_0}{4\pi}\int_{-d/2}^{d/2} \frac{-q_0\omega \sin[\omega(t - \mathscr{r}/c)]\,\hat{\mathbf{z}}}{\mathscr{r}}\,dz. \qquad \text{-------------- [13]}$$

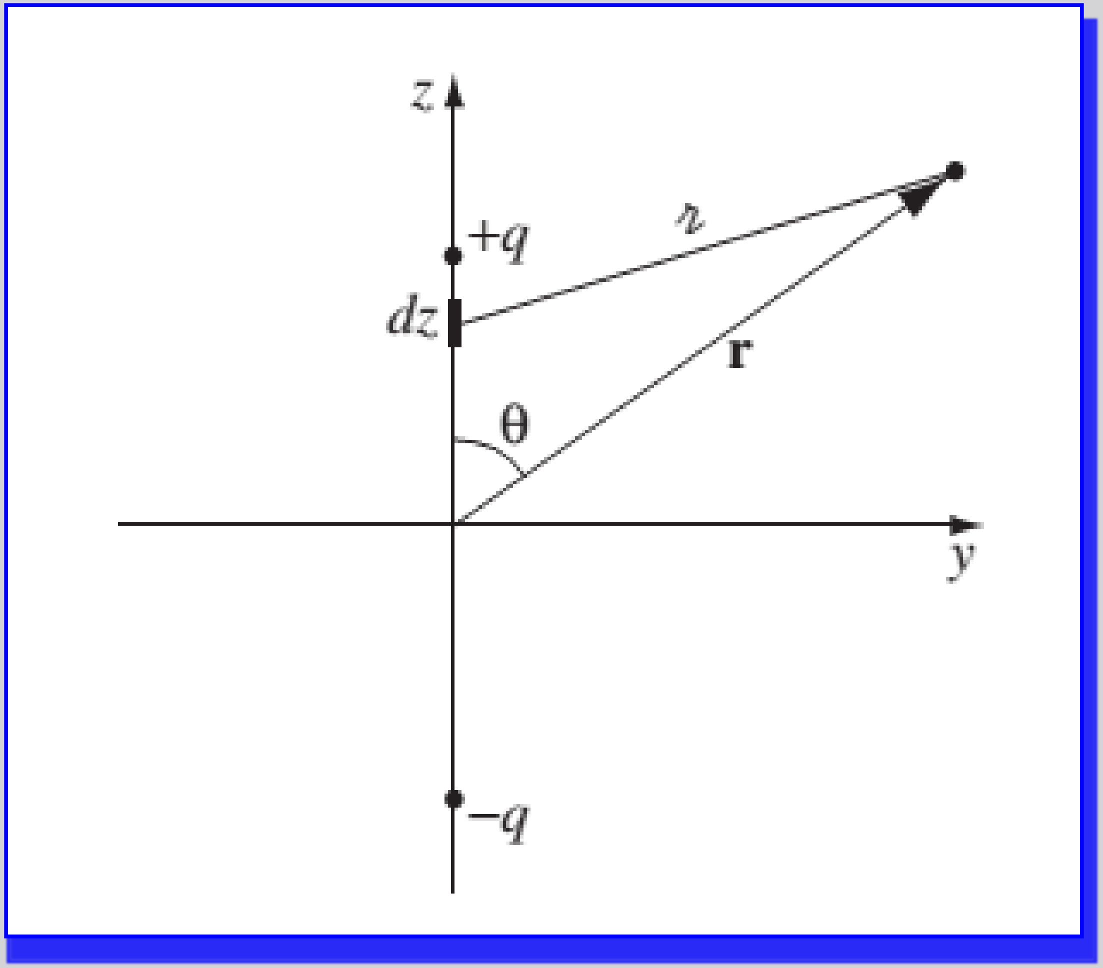


FIGURE 3:Representation of a current flowing wire **[REF-1]

Since the integration itself establishes a factor of *d*, we can, to first order, replace the integrand by its value at the center:

$$\boxed{\mathbf{A}(r, \theta, t) = -\frac{\mu_0 p_0 \omega}{4\pi r} \sin[\omega(t - r/c)]\, \hat{\mathbf{z}}.}$$ ------------- [14]

Calculation for Electric and Magnetic Fields

❖We know that the gradient of a scalar potential in terms of polar form Can be written as:

$$\boxed{\nabla V = \frac{\partial V}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{\boldsymbol{\theta}}}$$ ------------- [15]

$$\nabla V = -\frac{p_0\omega}{4\pi\epsilon_0 c}\left\{\cos\theta\left(-\frac{1}{r^2}\sin[\omega(t-r/c)] - \frac{\omega}{rc}\cos[\omega(t-r/c)]\right)\hat{\mathbf{r}} - \frac{\sin\theta}{r^2}\sin[\omega(t-r/c)]\,\hat{\boldsymbol{\theta}}\right\}$$

$$\nabla V \cong \frac{p_0\omega^2}{4\pi\epsilon_0 c^2}\left(\frac{\cos\theta}{r}\right)\cos[\omega(t-r/c)]\,\hat{\mathbf{r}}.$$ ------------- [16]

(we left the first and last terms, in accordance with approximation 3.) similarly,

$$\frac{\partial \mathbf{A}}{\partial t} = -\frac{\mu_0 p_0 \omega^2}{4\pi r}\cos[\omega(t-r/c)](\cos\theta\,\hat{\mathbf{r}} - \sin\theta\,\hat{\boldsymbol{\theta}}),$$ ------------- [17]

Therefore-

$$\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t} = -\frac{\mu_0 p_0 \omega^2}{4\pi}\left(\frac{\sin\theta}{r}\right)\cos[\omega(t - r/c)]\,\hat{\boldsymbol{\theta}}.$$

------------- [18]

meanwhile

$$\nabla \times \mathbf{A} = \frac{1}{r}\left[\frac{\partial}{\partial r}(r A_\theta) - \frac{\partial A_r}{\partial \theta}\right]\hat{\boldsymbol{\phi}}$$

------------- [19]

$$\nabla \times \mathbf{A} = -\frac{\mu_0 p_0 \omega}{4\pi r}\left\{\frac{\omega}{c}\sin\theta\cos[\omega(t - r/c)] + \frac{\sin\theta}{r}\sin[\omega(t - r/c)]\right\}\hat{\boldsymbol{\phi}}.$$

------------- [20]

The second term is again eradicated accordance approximation 3, so

$$\mathbf{B} = \nabla \times \mathbf{A} = -\frac{\mu_0 p_0 \omega^2}{4\pi c}\left(\frac{\sin\theta}{r}\right)\cos[\omega(t - r/c)]\,\hat{\boldsymbol{\phi}}. \quad \text{-------------- [21]}$$

❖ **Equations 18 and 21 representing the equation monochromatic fields equations of waves of frequency ω traveling in the radial direction at the speed of light. The fields are in same phase, mutually perpendicular, and transverse; the ratio of their amplitudes is $E_0/B_0 = c$.**

❖ **All of which is specifically what we suppose for electromagnetic waves in vacuum.**

❖ **These are in fact *spherical* waves, not plane waves, and their amplitude decreases like $1/r$ as they progress.**

❖ However for large *r*, they are approximately plane over small areas—just as the surface of the earth is sensibly flat, nearby.)

Poynting Vector and Radiated Power

❖ We know that the energy radiated by an oscillating electric dipole is obtained by the Poynting vector and the Poynting Vector in this case is defined as:

$$\mathbf{S}(\mathbf{r}, t) = \frac{1}{\mu_0}(\mathbf{E} \times \mathbf{B}) = \frac{\mu_0}{c}\left\{\frac{p_0\omega^2}{4\pi}\left(\frac{\sin\theta}{r}\right)\cos[\omega(t - r/c)]\right\}^2 \hat{\mathbf{r}}.$$

------------- [22]

❖ The intensity is determined by averaging (in time) over a full cycle and that is:

$$\langle \mathbf{S} \rangle = \left(\frac{\mu_0 p_0^2 \omega^4}{32\pi^2 c} \right) \frac{\sin^2\theta}{r^2}\,\hat{\mathbf{r}}.$$

------------- [23]

❖ It is observe that there is no radiation along the *axis* of the dipole (since sin θ = 0); the intensity profile take s the form of a donut, with its maximum in the equatorial plane (mentioned in Fig. 4).

❖ The total power radiated is obtained by integrating S over a sphere of radius *r*:

$$\langle P \rangle = \int \langle \mathbf{S} \rangle \cdot d\mathbf{a} = \frac{\mu_0 p_0^2 \omega^4}{32\pi^2 c} \int \frac{\sin^2\theta}{r^2} r^2 \sin\theta \, d\theta \, d\phi = \frac{\mu_0 p_0^2 \omega^4}{12\pi c}.$$

------------- [24]

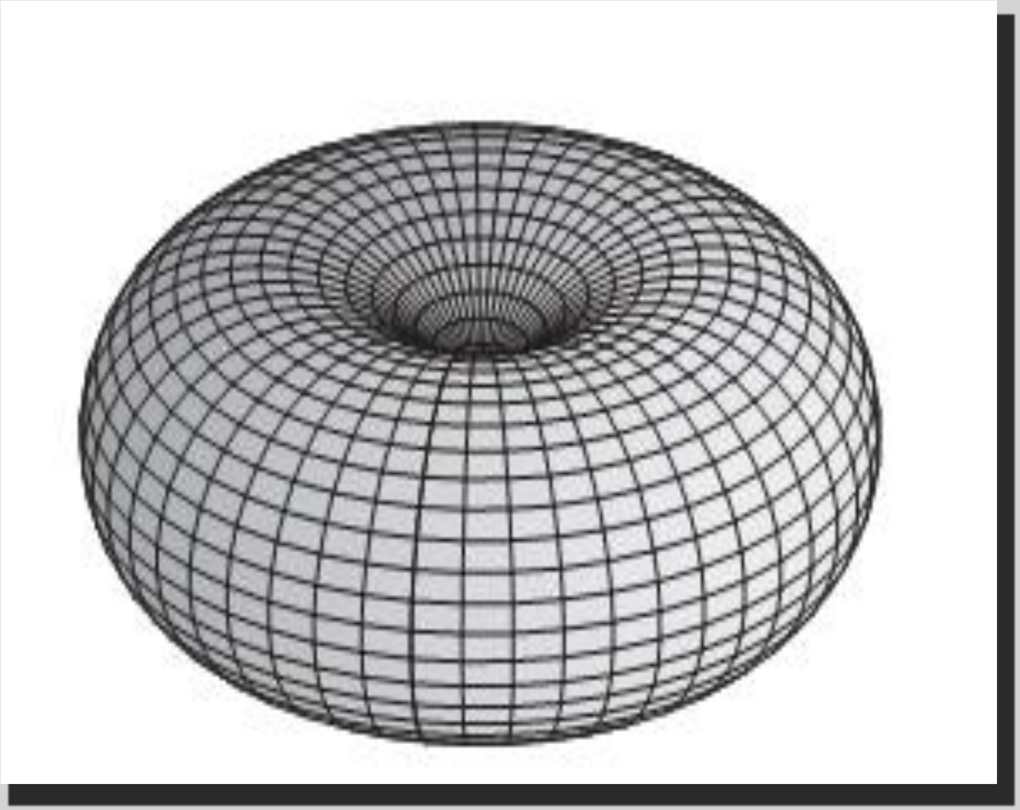
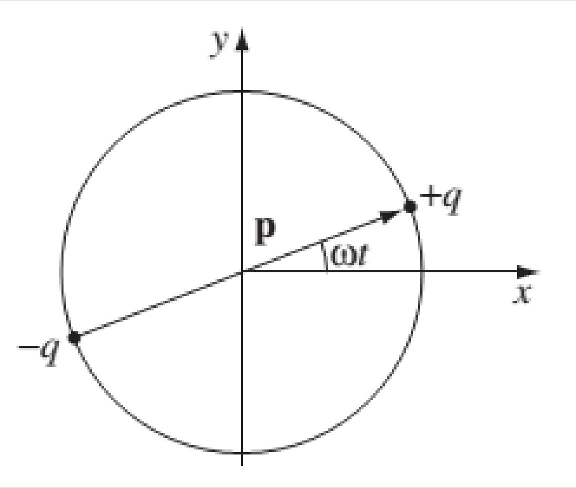


FIGURE 4: Representation of radiated power through a sphere of radius r . **[REF-1]

Numerical problems

1. Check that the retarded potentials of an oscillating dipole (Eqs. 11 and 14) satisfy the Lorenz gauge condition. Do *not* use approximation 3.
2. A rotating electric dipole can be thought of as the superposition of two oscillating dipoles, one along the x axis and the other along the y axis (Fig. 5), with the latter out of phase by 90°:

$$\mathbf{p} = p_0[\cos(\omega t)\,\hat{\mathbf{x}} + \sin(\omega t)\,\hat{\mathbf{y}}].$$



Using the principle of superposition and Eqs. 18 and 21, find the fields of a rotating dipole. Also find the Poynting vector and the intensity of the radiation. Sketch the intensity profile as a function of the polar angle θ, and calculate the total power radiated. Does the answer seem reasonable? (Note that power, being *quadratic* in the fields, does *not* satisfy the superposition principle. In this instance, however, it *seems* to. How do you account for this?)

References:

1. *Introduction to Electrodynamics, David J. Griffiths*

2. *Elements of Electromagnetics, 2nd edition by M N O Sadiku.*

3. *Engineering Electromagnetics by W H Hayt and J A Buck.*

4. *Elements of Electromagnetic Theory & Electrodynamics, Satya Prakash*

- **For any query/ problem contact me on whatsapp group or mail on me**

  E-mail: arvindkumar@mgcub.ac.in

- **Next *** Magnetic Dipole Radiation and Numerical Problems.**

Thank you