Business Research Methods

Course Code – MGMT4013

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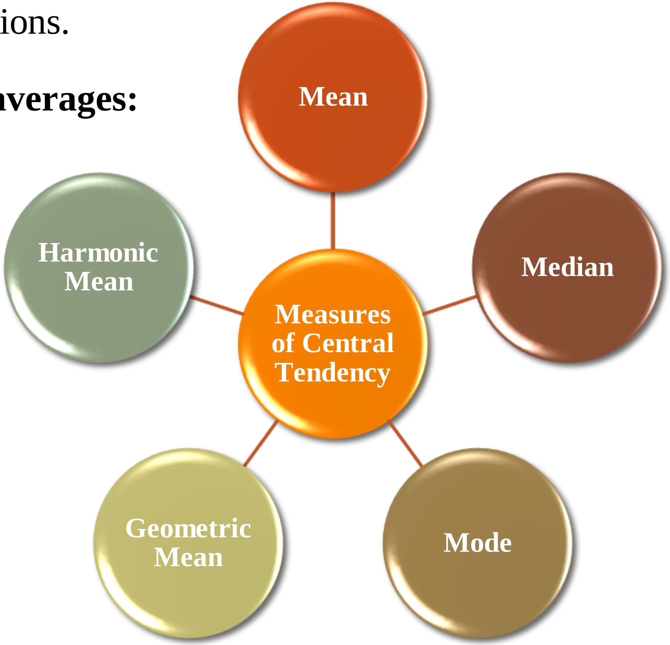
Content

- **Measures of Central Tendency and their merits and demerits**

❑ Measures of Central Tendency

This central value is called a measure of central tendency or an average or a measure of locations.

There are five averages:



Mean, median and mode are called **simple averages** and the other two averages geometric mean and harmonic mean are called **special averages**.

❑ Measures of Central Tendency Cont…

The meaning of average is nicely given in the following definitions.

- A measure of central tendency is a typical value around which other figures congregate.
- An average stands for the whole group of which it forms a part yet represents the whole.
- One of the most widely used set of summary figures is known as measures of location.

❑ Measures of Central Tendency Cont…

✓ **Characteristics for a good or an ideal average :**

The following properties should possess for an ideal average:

1. It should be rigidly defined.
2. It should be easy to understand and compute.
3. It should be based on all items in the data.
4. Its definition shall be in the form of a mathematical formula.
5. It should be capable of further algebraic treatment.
6. It should have sampling stability.
7. It should be capable of being used in further statistical computations or processing.

1. Arithmetic mean or mean :

Arithmetic mean or simply the mean of a variable is defined as the sum of the observations divided by the number of observations.

For Ungrouped or Raw Data

If the variable x assumes n values $x_1, x_2 \ldots x_n$ then the mean, x, is given by :

$$\bar{x} = \frac{x_1 + x_2 + x_3 + \ldots + x_n}{n}$$

$$= \frac{1}{n}\sum_{i=1}^{n} x_i$$

- **Short-Cut method :**

Under this method an assumed or an arbitrary average (indicated by A) is used as the basis of calculation of deviations from individual values.

The formula is :

$$\bar{x} = A + \frac{\sum d}{n}$$

where, A = the assumed mean or any value in x.
d = the deviation of each value from the assumed mean.

1. Arithmetic mean or mean Cont…

Grouped Data :

The mean for grouped data is obtained from the following formula:

$$\bar{x} = \frac{\sum fx}{N}$$

Where, *x* = the mid-point of individual class
f = the frequency of individual class
N = the sum of the frequencies or total frequencies.

- **Short-Cut method :**

$$\bar{x} = A + \frac{\sum fd}{N} \times c$$

Where, $d = \frac{x - A}{c}$

A = any value in x
N = total frequency
c = width of the class interval

1. Arithmetic mean or mean Cont…

- ✓ **Direct Method for Grouped Data**

Formula is :

$$\bar{x} = \frac{\sum fx}{N}$$

- ✓ **Short-cut Method for Grouped Data**

Formula is :

$$\bar{x} = A + \frac{\sum fd}{N}$$

1. Arithmetic mean or mean Cont…

- **Merits and demerits of Arithmetic mean :**

Merits	Demerits
It is rigidly defined.	It cannot be obtained by inspection nor located through a frequency graph.
It is easy to understand and easy to calculate.	It cannot be in the study of qualitative phenomena not capable of numerical measurement i.e. Intelligence, beauty, honesty etc.,
If the number of items is sufficiently large, it is more accurate and more reliable.	It can ignore any single item only at the risk of losing its accuracy.
It is a calculated value and is not based on its position in the series.	It is affected very much by extreme values.
It is possible to calculate even if some of the details of the data are lacking.	It cannot be calculated for open-end classes.
Of all averages, it is affected least by fluctuations of sampling.	It may lead to fallacious conclusions, if the details of the data from which it is computed are not given.
It provides a good basis for comparison.	

✓ Weighted Arithmetic mean :

The average whose component items are being multiplied by certain values known as "weights" and the aggregate of the multiplied results are being divided by the total sum of their "weight".

If $x_1, x_2 \ldots x_n$ be the values of a variable x with respective weights of $w_1, w_2 \ldots w_n$ assigned to them, then

Weighted A.M = $$\bar{x}_w = \frac{w_1x_1 + w_2x_2 + \ldots. + w_nx_n}{w_1 + w_2 + \ldots. + w_n} = \frac{\sum w_ix_i}{\sum w_i}$$

- **Uses of the weighted mean:**

Weighted arithmetic mean is used in:

a. Construction of index numbers.

b. Comparison of results of two or more universities where number of students differ.

c. Computation of standardized death and birth rates.

2. Harmonic mean (H.M) :

Harmonic mean of a set of observations is defined as the reciprocal of the arithmetic average of the reciprocal of the given values. If $x_1, x_2 \ldots\ldots x_n$ are n observations,

$$\text{H.M} = \frac{n}{\sum_{i=1}^{n}\left(\frac{1}{x_i}\right)}$$

For a frequency distribution,

$$HM = \frac{N}{\sum_{i=1}^{n} f\left(\frac{1}{x_i}\right)}$$

2. Harmonic mean (H.M) Cont…

- **Merits and demerits of Harmonic mean :**

Merits	Demerits
It is rigidly defined.	It is not easily understood.
It is defined on all observations.	It is difficult to compute.
It is amenable to further algebraic treatment.	It is only a summary figure and may not be the actual item in the series
It is the most suitable average when it is desired to give greater weight to smaller observations and less weight to the larger ones.	It gives greater importance to small items and is therefore, useful only when small items have to be given greater weightage.

3. Geometric Mean :

The geometric mean of a series containing n observations is the n^{th} root of the product of the values. If $x_1, x_2, \ldots, x_n$ are observations then

For Ungrouped Data :

Formula is :

$$G.M = \sqrt[n]{x_1 . x_2 \ldots x_n}$$

$$= (x_1 . x_2 \, .. x_n)^{1/n}$$

$$\log GM = \frac{1}{n} \log(x_1 . x_2 \, .. x_n)$$

$$= \frac{1}{n} (\log x_1 + \log x_2 + .. + \log x_n$$

$$= \frac{\sum \log x_i}{n}$$

$$GM = \text{Antilog} \; \frac{\sum \log x_i}{n}$$

For Grouped Data :

Formula is :

$$GM = \text{Antilog} \left[\frac{\sum f \log x_i}{N} \right]$$

3. Geometric Mean Cont…

- **Merits and demerits of Harmonic mean :**

Merits	Demerits
It is rigidly defined.	It cannot be used when the values are negative or if any of the observations is zero
It is based on all items.	It is difficult to calculate particularly when the items are very large or when there is a frequency distribution.
It is very suitable for averaging ratios, rates and percentages.	It brings out the property of the ratio of the change and not the absolute difference of change as the case in arithmetic mean.
It is capable of further mathematical treatment.	The GM may not be the actual value of the series.
Unlike AM, it is not affected much by the presence of extreme values	

4. Median :

The median is that value of the variate which divides the group into two equal parts, one part comprising all values greater, and the other, all values less than median.

For Ungrouped or Raw data :

- Arrange the given values in the increasing or decreasing order.
- If the number of values are odd, median is the middle value.
- If the number of values are even, median is the mean of middle two values.

Formula is :

$$\textbf{Median} = \text{Md} = \left(\frac{n+1}{2}\right)^{\text{th}} \text{item.}$$

4. Median Cont…

For Grouped Data:

- In a grouped distribution, values are associated with frequencies.
- Grouping can be in the form of a discrete frequency distribution or a continuous frequency distribution.
- cumulative frequencies have to be calculated to know the total number of items.

Cumulative frequency : (cf)

Cumulative frequency of each class is the sum of the frequency of the class and the frequencies of the pervious classes, i.e.. adding the frequencies successively, so that the last cumulative frequency gives the total number of items.

Discrete Series:

Step1: Find cumulative frequencies.

Step2: Find $\left(\frac{N+1}{2}\right)$

Step3: See in the cumulative frequencies the value just greater than $\left(\frac{N+1}{2}\right)$

Step4: Then the corresponding value of x is median.

4. Median Cont…

Continuous Series:

The steps given below are followed for the calculation of median in continuous series.

Step1: Find cumulative frequencies.

Step2: Find $\left(\frac{N+1}{2}\right)$

Step3: See in the cumulative frequency the value first greater than $\left(\frac{N+1}{2}\right)$,

Then the corresponding class interval is called the Median class.

Then apply the formula :

Median = $l + \frac{\frac{N}{2} - m}{f} \times c$

4. Median Cont…

Where, l = Lower limit of the median class

m = cumulative frequency preceding the median

c = width of the median class

f =frequency in the median class.

N=Total frequency.

Note :If the class intervals are given in inclusive type convert them into exclusive type and call it as true class interval and consider lower limit in this.

4. Median Cont…

- **Merits and demerits of median :**

Merits	Demerits
Median is not influenced by extreme values because it is a positional average.	A slight change in the series may bring drastic change in median value.
Median can be calculated in case of distribution with opened intervals.	In case of even number of items or continuous series, median is an estimated value other than any value in the series.
Median can be located even if the data are incomplete.	It is not suitable for further mathematical treatment except its use in mean deviation.
Median can be located even for qualitative factors such as ability, honesty etc.	It is not taken into account all the observations.

5. Mode :

Mode refers to that value in a distribution, which occur most frequently.
It is an actual value, which has the highest concentration of items in and around it.

According to Croxton and Cowden " The mode of a distribution is the value at the point around which the items tend to be most heavily concentrated. It may be regarded at the most typical of a series of values".

Its importance is very great in marketing studies where a manager is interested in knowing about the size, which has the highest concentration of items.

Computation of the mode for Ungrouped or Raw Data :

For ungrouped data or a series of individual observations, mode is often found by mere inspection.

Example : 2 , 7, 10, 15, 10, 17, 8, 10, 2

So, Mode = $M_0 = 10$

In some cases the mode may be absent while in some cases there may be more than one mode.

5. Mode Cont…:

Computation of the mode for Grouped:

For Discrete distribution, see the highest frequency and corresponding value of X is mode.

Continuous distribution :

The highest frequency then the corresponding value of class interval is called the modal class.

Then apply the formula:

$$\textbf{Mode} = M_0 = l + \frac{\Delta_1}{\Delta_1 + \Delta_2} \times C$$

l = Lower limit of the model class

$\Delta_1 = f_1 - f_0$

$\Delta_2 = f_1 - f_2$

f_1 = frequency of the modal class

f_0 = frequency of the class preceding the modal class

f_2 = frequency of the class succeeding the modal class

5. Mode Cont…:

The above formula can also be written as:

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times c$$

- **Merits and demerits of mode:**

Merits	Demerits
It is easy to calculate and in some cases it can be located mere inspection	It is not based on all observations.
Mode is not at all affected by extreme values.	It is not capable of further mathematical treatment.
It can be calculated for open-end classes.	Mode is ill-defined generally, it is not possible to find mode in some cases.
It is usually an actual value of an important part of the series.	As compared with mean, mode is affected to a great extent, by sampling fluctuations.
In some circumstances it is the best representative of data.	It is unsuitable in cases where relative importance of items has to be considered.

❑ Sources

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