Annexure-II

महात्मा गाँधी केन्द्रीय विश्वविद्यालय
MAHATMA GANDHI CENTRAL UNIVERSITY
(Established by an Act of Parliament)
Chanakya Parisar, Zila School Campus, Motihari
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# Ph.D. Programme

## (Effective from Academic Year: 2022 -23)

## Ph.D. Course Work (Mathematics)

## Detailed Course Outline

## Semester-I

## Compulsory Courses

**Course Code:** MATH5001
**Course Title:** Research Methodology and Publication Ethics
**Credits:** 4 (total contact hr. 60)
**Course Objectives:** This course focusing on basics of philosophy of science and ethics, research integrity, publication ethics. Hands-on-sessions are designed to identify research misconduct and predatory publications. Indexing and citation databases, open access publications, research metrics (citation, h-index, Impact Factor, etc.) and plagiarism tools will be introduced in this course. Also this course is to ensure that a student learns basis of scientific research and to develop understanding of computer and its few basic applications.
**Course Contents:**
**Unit I:** Introduction to philosophy: definition, nature and scope, concept, branches, Ethics: definition, moral philosophy, nature of moral judgments and reactions, Ethics with respect to science and research, Intellectual honesty and research integrity, Scientific misconducts: Falsification, Fabrication and plagiarism (FFP), Redundant publications: duplicate and overlapping publications, salami slicing, Selective reporting and misrepresentation of data Publication ethics: Definition, introduction and importance, Best practices / standards setting initiatives and guidelines: COPE, WAME etc., Conflicts of interest, Publication misconduct: Definition, concept, problems that lead to unethical behavior and vice versa, types, Violation of publication ethics, authorship and contributor ship, Identification of publication misconduct, complaints and appeals, Predatory publishers and journals.
**Unit II:** OPEN ACCESS PUBLISHING: Open access publication and initiatives, SHERPA/RoMEO online resource to check publisher copyright & self-archiving policies, Software tool to identify predatory publications developed by SPPU, Journal finder/journal suggestion tools

viz. JANE, Elsevier Journal Finder, Springer Journal Suggester, etc., PUBLICATION MISCONDUCT : Group Discussions, Subject specific ethical issues, FFP, authorship, Conflicts of interest, Complaints and appeals: examples and fraud from India and abroad, Software tools, Use of plagiarism software like Turnitin, Urkund and other source software tools, DATABASES AND RESEARCH METRICS : Databases , Indexing databases, Citation databases: Web of science, Scopus, etc., Research Metrics, Impact Factor of journal as per journal Citation Report, SNIP, SJR, IPP, Cite Score, Metrics: h-index, g index, i10 index, altmetrics.

**Unit – III: Introduction to Research Methodology:** Meaning and importance of research, Selection and formulation of research problem, Research design, Developing a research plan – exploration, description, diagnosis, experimentation, Determining experimental and sample designs, Analysis of literature review, Hypothesis, Development of working hypothesis, Research methods: scientific method *vs* arbitrary method.

**Unit – IV: Data Collection and Analysis:** Sources of data, Methods of collecting data: observation, field investigations, Direct studies, Sampling methods, Data processing and analysis strategies, Graphical representation, Descriptive analysis, Inferential analysis, Correlation analysis, Least-square methods, Errors classification and analysis, **Scientific Writing:** Language of science and technology, Technical presentations: design and delivery, Collecting materials for research, Organization of research paper/dissertation/thesis, Citation index, Preparation of a project proposal, Use of E-journal and E-Library, **Basic Mathematical Software & Computer Skills:** LATEX, A brief introduction to MATLAB and Mathematica, Scientific graphic design softwares.

**Reference Books:**


1. K. Muralidhar, A. Ghosh and A.K. Singhvi, Ethics in Science Education, Research and Governance, Indian National Science Academy, New Delhi, 2019.
2. J.G. D’Angelo, Ethics in Science: Ethical Misconduct in Scientific Research, 2$^{nd}$ ed., CRC Press, 2019.
3. M. Mauthner, M. Birch, J. Jessop and T. Miller, Ethics in Qualitative Research, Sage Publications, New Delhi, 2005.
4. Rose Wiles, What are Qualitative Research Ethics? Bloomsbury Academic, New Delhi, 2013.
5. M. Hammersley and A. Traianou, Qualitative Research: Controversies and Contexts, Sage Publications, New Delhi, 2012.
6. B. L. Garg., R. Karadia, F. Agarwal, and U. K. Agarwal, An Introduction to Research Methodology, RBSA Publishers, 2002.
7. C. R. Kothari and G. Garg, Research Methodology: Methods and Techniques, 4$^{th}$ ed., New Age International Publishers, New Delhi, 2019.
8. S. C. Sinha and A. K. Dhiman, Research Methodology, Ess Publications, 2002.
9. W. M. K. Trochim, Research Methods: The Concise Knowledge Base, Atomic Dog Publishing, 2005.
10. B. Gastel and R. A. Day, How to Write and Publish a Scientific Paper, 8th ed., Greenwood, An Imprint of ABC-CLIO, LLC, Santa Barbara, California , 2016.

11. B. Latour, and Woolgar, Laboratory Life: The Construction of Scientific Facts, 2nd ed., Princeton: Princeton University Press , 1986.
12. N. M. Downine, Basic Statistical Methods, New York, Harper and Health Row Publishers, 1974.
13. Harry. Frank, Statistics: Concepts and Applications, Cambridge, Althoen, Steven Cambridge University, 1994.
14. P. K. Sinha, Computer Fundamentals, BPB Publications, New Delhi, 1992.

**Course Code: MATH5002**
**Course title:** Review Writing and Seminar
**Course Credits: 2** (total contact hr. 30)
**Objective:** The objective of this course would be to ensure that the student learns the aspects of the review writing and seminar presentation. Herein the student shall have to write a review of existing scientific literature (on the topic of his /her choice lying within the elective courses opted by him / her) with simultaneous identification of knowledge gaps that can be addressed through future work.

The evaluation criteria for "Review Writing and Seminar" shall be as follows:

| **S. No.** | **Criteria** | **Marks** |
|---|---|---|
| 1. | Review of literature | 25 |
| 2. | Identification of gaps in knowledge | 15 |
| 3. | Content of presentation | 25 |
| 4. | Presentation skills | 20 |
| 5. | Handling of queries | 15 |
| | **Total** | **100** |

**Course Code: MATH5003**
**Course title:** Foundations of Advanced Mathematics
**Course Credits: 4** (total contact hr. 60)
**Objective:** The objective of this course is to introduce students the basic ideas from Algebra, Analysis, Geometry and Differential equations so that they can embark on further studies and research in various areas of pure and applied mathematics.
**Course Contents:**
**Unit I:** Composition series, solvable groups, nilpotent groups, Jordan-Holder theorem. Extension fields, finite and algebraic extensions, splitting fields, fundamental theorem of field theory.
**Unit II:** Multivariable calculus: Differentiability of functions from $\mathbb{R}^n$ to $\mathbb{R}^m$, partial derivatives and differentiability, directional derivatives, chain rule, inverse function theorem, implicit function theorem.

**Unit III:** Normed linear spaces, Banach spaces and examples, bounded and continuous linear transformations. $B(X, Y)$, dual of $X$, Bounded linear functionals, Hahn-Banach extension theorem, adjoint operator, Hilbert space, Examples of orthonormal basis in Hilbert spaces, operators on Hilbert space.

**Unit-IV:** Introduction to differential equations, Picard's existence and uniqueness theorem, existence and uniqueness of initial value problem with examples, system of linear differential equations, series solution, Bessel and Legendre equations, Wave, Heat and Laplace equations and their solutions by the methods of separation of variables and integral transforms.

**Reference Books**:


1. Joseph A. Gallian, Contemporary Abstract Algebra 4th ed., Narosa Publishing House, New Delhi, 1999.
2. I. N. Herstein, Topics in Algebra, Wiley Eastern Ltd., New Delhi, 1975.
3. W. Rudin, Principles of Mathematical Analysis, 3rd ed., McGraw Hill, 1986.
4. T.M. Apostol, Mathematical Analysis, 2nd ed., Narosa Publishing House, 2002.
5. E. Kreyszig, Introductory Functional Analysis with Applications, John Wiley & Sons, 2006.
6. A. E. Taylor, Introduction to Functional Analysis, John Wiley, 1958.
7. Walter Rudin, Functional Analysis, Tata McGraw Hill, 2010.
8. George F. Simmons, Differential Equations with Application and Historical Notes, McGraw Hill Higher Education, 1979.
9. Shepley L. Ross, Differential Equations, 3rd ed., Wiley publication, 2016.
10. L. C. Evans, Partial Differential Equations, Graduate Studies in Mathematics, Vol. 19, AMS, 1999.
11. K. Sankara Rao, Introduction to Partial Differential Equations, PHI Learning, 2010.


## Elective Courses

**Course Code: MATH5101**

**Course title:** Wavelets and Wavelet Transforms

**Course Credits: 3** (total contact hr. 45)

**Course Objectives:** The objective of the course is to study basic concepts of the short-time Fourier transform, wavelets, wavelet transforms, Multi Resolution Analysis (MRA) and construction of wavelets. The outcome of this course is to help the students to do research in wavelet analysis as well as in allied areas.

**Course Contents:**

**Unit - I**

Signals and systems, The short-time Fourier transform (STFT), Elementary properties of STFT.

**Unit - II**

Definition and examples of wavelets, Wavelet transforms and its basic properties, Frames and frame operators, Orthonormal wavelets.

**Unit - III**

Definition of multi resolution analysis and examples, Properties of scaling functions and orthonormal wavelet bases.

**Unit - IV**

Construction of wavelets, Cardinal B-splines, Franklin wavelet, Battle-Lemarie wavelet, Daubechies' wavelets.

**Reference Books:**


1. Lokenath Debnath and Firdous Ahmad Shah, Wavelet Transforms and their Applications, $2^{nd}$ ed., Birkhauser, 2015.
2. Ingrid Daubechies, Ten Lectures on Wavelets, SIAM: Society for Industrial and Applied Mathematics, 1992.
3. C.K. Chui, An Introduction to Wavelets, Academic Press, 1992.
4. A. Boggess and F. J. Narcowich, A First Course in Wavelets with Fourier Analysis, Wiley, $2^{nd}$ ed., 2009.
5. Eugenio Hernandez and Guido L. Weiss, A First Course on Wavelets, CRC Press, 1996.
6. David F. Walnut, An Introduction to Wavelet Analysis, Birkhauser, 2004.
7. P. Wojtaszczyk, A Mathematical Introduction to Wavelet, Cambridge University Press, 1997.


**Course Code: MATH5102**

**Course title:** Distribution Theory

**Course Credits: 3** (total contact hr. 45)

**Course Objectives:** The objective of the course is to introduce students to the basic concepts of Distributions to prepare them for further studies and research in Distribution Theory and their applications in allied areas.

**Course Contents:**

**Unit - I**

Test functions and distributions, Basic properties of distributions, Convergence of distributions.

**Unit - II**

Differentiation of distributions, Distributions of compact support, Convolution of distributions, Regularization of distributions, Applications to differential equations.

**Unit - III**

The space of rapidly decreasing functions S, The space of tempered distributions S', Fourier transformation in S, Fourier transformation in S', Convolution theorem in S', Fourier transformation in E'.

**Unit - IV**

Sobolev space $W^{\{m,p\}}$, Sobolev space $H^s$, Product and convolution in $H^s$, Weak solutions boundary value problems.

**Reference Books:**


1. M.A. Al-Gwaiz, Theory of Distributions, CRC Press, 1992.

2. A. H. Zemanian, Distribution Theory and Transform Analysis: An Introduction to Generalized Functions with Applications, Dover Publications Inc, 2003.
3. A. H. Zemanian, Generalized Integral Transforms, Dover Publications Inc, 1987.
4. R. S. Pathak, A Course in Distribution Theory and Applications, Narosa, 2001.
5. R. Strichartz, A Guide to Distribution Theory and Fourier Transforms, World Scientific, 2003.

**Course Code: MATH5103**

**Course title:** Operator Theory

**Course Credits: 3** (total contact hr. 45)

**Course Objective:** Objective of this course is to introduce operators on Hilbert space, compact linear operators, spectral theory, spectral properties of compact operators, spectral theory of self-adjoint operators and Banach algebra.

**Course Contents:**

**Unit - I**

Operators on Hilbert Spaces, Adjoint operator, Self-adjoint operators, Normal operators, Unitary operators, Projection operators, Spectrum of bounded operators, Resolvent, Spectral properties of bounded operators.

**Unit - II**

Compact linear operators, Basic properties, Adjoint of compact operators, Spectral properties of compact operators, Fredholm alternative.

**Unit - III**

Spectral theory of self-adjoint operators, Positive operators and their properties, Spectral representation of a self-adjoint compact operator, Spectral family of a self-adjoint operator and its properties, Spectral representation of a self-adjoint operator, Continuous functions of self-adjoint operators.

**Unit - IV**

Banach algebra with examples, Regular and singular elements, Topological division of zero, Spectral mapping theorem for polynomials and spectral radius formula, Ideals in Banach algebra, Commutative Banach algebra with examples, Gelfand transform, Maximal-ideal space with examples.

**Reference Books:**

1. Walter Rudin, Functional Analysis, Tata McGraw-Hill, 2010.
2. G. Bachman and L. Narici, Functional analysis, Academic Press, New York, 1998.
3. B. V. Limaye, Functional Analysis, 3rd ed., New Age International Ltd., 2014.
4. G. F. Simmons, An Introduction to Topology and Modern Analysis, Tata McGraw-Hill, 2004.
5. J. B. Conway, A Course in Operator Theory: A Graduate Studies in Mathematics, Springer, 1985.
6. M. Schechter, Principles of Functional Analysis, 2nd ed., American Mathematical Society, 2001.

7. E. Kreyszig, Introductory Functional Analysis with Applications, John Wiley & Sons, India, 2006.

**Course Code: MATH5104**

**Course title:** Advanced Topology

**Course Credits: 3** (total contact hr. 45)

**Course Objective:** This is a course on point-set topology. It is designed in such a way that the students can have deep knowledge in general topology and can understand further deeper topics in differential/algebraic topologies and their allied areas.

**Course Contents:**

**Unit - I**

Topological space, Quotient topology, Product topology, Metric topology, Topological manifolds, Connectedness, Compactness, Countability and Separation axioms.

**Unit - II**

Metrizable spaces and metrization theorems, One-point compactification, Stone-Cěch compactification, Paracompact space and its characterizations.

**Unit - III**

Uniform spaces, Weak uniformity, Uniformizability, Completion of uniform spaces.

**Unit - IV**

Function spaces, Point-wise and uniform convergence, Compact open topology, Stone-Weierstrass theorem, Arzela-Ascoli’s theorem.

**Reference Books**:

1. S. Willard, General Topology, Addison-Wesley, 1970.
2. S. W. Davis, Topology, Tata McGraw-Hill, 2006.
3. K. D. Joshi, Introduction to General Topology, Wiley Eastern, Delhi, 1986.
4. J. R. Munkres, Topology: A First Course, Prentice-Hall of India, New Delhi, 1975.
5. James Dugundji, Topology, Universal Book Stall, New Delhi, 1990.

**Course Code: MATH5105**

**Course title:** Category Theory

**Course Credits: 3** (total contact hr. 45)

**Course Objective:** Category theory is a language for most of the modern mathematics. It is also very convenient to express mathematical concepts using category theory. It has a vast application in computer science and other areas also. The objective of this course is to teach students the basic knowledge of category theory.

**Course Contents:**

**Unit - I**

Categories, Functors and natural transformations, Monics, epis and zeors, Construction on categories, Duality, Contravariant and opposite categories.

**Unit - II**

Products of categories, Functor categories, Comma categories, Universals and limits, Universal arrows, The Yoneda lemma, Coproducts and colimits, Products and limits, Adjoints, Examples of adjoints, Reflective subcatgories.

**Unit - III**

Equivalence of categories, Adjoints for preorders, Cartesian closed categories, Limits, Creation of limits, Limits by products and equailizer, Freyd's adjoint functor theorem, Special adjoint functor theorem, Adjoints in topology.

**Unit - IV**

Monads and algebras, Monads in a category, Algebra for a monad, The comparison with algebras, Words and free semi groups, Free algebras for monads, Beck's theorem, Algebras are T-algebras, Compact Hausdorff spaces.

**Reference Books**:


1. S. MacLane, Categories for the Working Mathematician, Springer, 1971.
2. M. A. Arbib and E. G. Manes, Arrows, Structures and Functors: The Categorical Imperative, Academic Press, 1975.
3. H. Herrlich and G. E. Strecker, Category Theory, Allyn & Bacon, 1973.
4. J. Adamek, H. Herrlich and G. E. Streeker, Abstract and Concrete Categories, John Wiley, 1992.


**Course Code: MATH5106**

**Course title:** Number Theory & Cryptography

**Course Credits: 3** (total contact hr. 45)

**Course Objective:** The objective of this course is to introduce students to some of the basic ideas of number theory and cryptography. This course will introduce and illustrate different methods of proof in the context of elementary number theory and will apply some basic techniques of number theory to cryptography.

**Course Contents:**

**Unit - I**

Divisibility, Primes, Fundamental theorem of arithmetic, Modular arithmetic, Residue systems, Fermat's little theorem, Euler's generalization, Wilson's theorem, Chinese remainder theorem, Diophantine equations: solutions of $ax + by = c,\ x^2 + y^2 = z^2$, Representation of a positive integer as a sum of two and four squares.

**Unit - II**

Arithmetical functions: $\varphi(n), \mu(n), d(n), \sigma(n)$, Mobious inversion formula and its applications, Quadratic congruence, Congruence modulo powers of prime, Primitive roots and indices, Quadratic residues, Legendre and Jacobi symbols, Euler's criteria, Gauss's lemma about Legendre symbol, Quadratic law of reciprocity.

**Unit - III**

Secure communication, Symmetric and asymmetric cryptosystems, Block and stream ciphers, Affine linear block ciphers and their cryptanalysis.

Perfect secrecy, Vernamone time pad, Secure random bit generator, Linear feedback shift registers.

**Unit-IV**

Feistel cipher, DES, Modes of DES, AES, RSA, Diffie Hellman key exchange, Elgamal public key cryptosystem.

**Reference Books:**


1. David M. Burton, Elementary Number Theory, 6th ed., McGraw-Hill, New York, 2007.
2. G.A. Jones and J.M. Jones, Elementary Number Theory, Springer-Verlag, 1998.
3. Niven, S.H. Zuckerman and L.H. Montgomery, An Introduction to the Theory of Numbers, John Wiley, 1991.
4. Joseph H. Silverman, A Friendly Introduction to Number Theory, 4th ed., Pearson, 2012.
5. Thomas Koshy, Elementary Number Theory with Applications, 2nd ed., Academic Press, 2007.
6. T. Nagell, Introduction to Number Theory, AMS Chelsea Publishing, 2009.
7. G.H. Hardy and E.M. Wright, Introduction to the Theory of Numbers, 6th ed., Oxford University Press, 2008.
8. J.A. Buchmann, Introduction to Cryptography, 2nd ed., Springer 2003.
9. D.R. Stinson, Cryptography: Theory and Practice, CRC Press, Taylor & Francis, 2005.


**Course Code: MATH5107**

**Course title:** Fuzzy Topology

**Course Credits: 3** (total contact hr. 45)

**Course Objective:** The course is an introductory course on fuzzy topology. It is designed in such a way that the students can have basic knowledge in fuzzy topology including some of its categorical aspects.

**Course Contents:**

**Unit - I**

Fuzzy set theory: Basic definitions of fuzzy sets, Various operations on it, Representation of fuzzy sets through α- cuts, Backward and forward operators related to functions between sets, Zadeh extension principle, Fuzzy points, Belonging relations and quasi-coincidence, Some generalization of fuzzy sets including l-fuzzy sets, Introductory accounts of intuitionistic fuzzy sets and rough sets.

**Unit - II**

Fuzzy topology: Definition and examples of fuzzy topology and its various suggested modifications, Fuzzy continuity, Initial and final fuzzy topologies, Induced fuzzy topologies.

**Unit - III**

Fuzzy product topology, Fuzzy separation axioms, Connectedness and compactness, Fuzzy sobriety.

**Unit - IV**

Categorical aspects of fuzzy topology: Basic concepts of category theory, The category FTS of fuzzy topological spaces and its being a topological category, The relation between FTS and the category TOP of topological spaces, Some reflective and co-reflective subcategories of FTS and related categories.

**Reference Books**:


1. Y.-M. Liu and M.-K.Luo, Fuzzy Topology, World Scientific, 1997.
2. N. Palaniappan, Fuzzy Topology, Narosa Publishing House, 2002.


**Course Code: MATH5108**

**Course title:** Theory of Nonlinear Optimization

**Course Credits: 3** (total contact hr. 45)

**Course objective:**

Optimization problem arise in a variety of discipline. This course provides core concepts of continuous optimization and includes a thorough introduction to unconstrained and constrained optimization problems.

**Course Content:**

**Unit - I**

Introduction and problem formulation, Existence theorems for minimal points, Set of minimal Points, Application to approximation problems, Application to optimal control problems.

**Unit - II**

Generalized derivatives: directional derivative, Gateaux and Frechet derivatives, sub-differential, quasi-differential, Clarke derivative, Tangent cones: definition and properties, optimality conditions, A Lyusternik theorem.

**Unit - III**

Generalized Lagrange multiplier rule: necessary optimality conditions, Sufficient optimality conditions, Application to optimal control problems.

**Unit - IV**

Constrained and unconstrained optimization techniques, Some direct search and indirect search methods, Quadratic programming, Geometric programming.

**Reference Books:**


1. J. Nocedal and S.J. Wright, Numerical Optimization, Springer, 1999.
2. R. Fletcher, Practical Methods of Optimization, John Wiley, 1980.
3. A. Charnes, W.W. Cooper, Mathematical Models and Industrial Applications of Linear Programming, Vol. 1 & Vol. 2, Wiley, New York, 1961.
4. Johannes Jahn, Introduction to the Theory of Nonlinear Optimization, 3rd ed., Springer, 2007.
5. J. Jahn, Vector Optimization: Theory, Applications and Extensions, Springer, 2004.


**Course Code: MATH5109**

**Course title:** Fuzzy Sets and Application in Operations Research

**Course Credits: 3** (total contact hr. 45)

**Course Objective:** Operations research present state and future trends are analysed from the Fuzzy-sets-based methodologies point of view. Then, in a more particular perspective, some specific topics of fuzzy-sets-based models in operations research, such as fuzzy optimization, preference modelling, linguistic modelling and decision models as well as some other well-known operations research specific topics are also reviewed and prospected.

Course Content:

**Unit - I**

Basic concepts of fuzzy sets and fuzzy logic: fuzzy sets and their representations, Membership functions and their designing, Types of fuzzy sets, Operations on fuzzy sets, Convex fuzzy sets, Alpha-level cuts, Geometric interpretation of fuzzy sets, Linguistic variables, Possibility measure and distribution.

**Unit - II**

Fuzzy relations and fuzzy arithmetic: composition of fuzzy relations, Fuzzy numbers, Arithmetic operations on fuzzy numbers, Fuzzy reasoning, Fuzzy mapping rules and fuzzy implication rules, Fuzzy rule-based models for function approximation, Types of fuzzy rule-based models (the Mamdani, TSK, and standard additive models).

**Unit - III**

Fuzzy logic and probability theory: possibility versus probability, Probability of a fuzzy event, Baye's theorem for fuzzy events, Probabilistic interpretation of fuzzy sets, Fuzzy measure.

**Unit - IV**

Decision making in fuzzy environment: fuzzy decisions, Fuzzy linear programming, Fuzzy multi criteria analysis, Basic concept of multi objective and multi-level optimization, Linear fractional programming, Goal programming.

**Reference Books**:


1. J. Yen and R. Langari, Fuzzy Logic: Intelligence, Control and Information, Pearson Education, 2003.
2. G. J. Klir and B. Yuan, Fuzzy Sets and Fuzzy Logic: Theory and Applications, Prentice-Hall of India, 1997.
3. H.J. Zimmermann, Fuzzy Set Theory and its Applications, Kluwer Academic Publ., 2001.
4. F. S. Hiller and G. J. Leiberman, Introduction to Operations Research, 9th ed., McGraw-Hill International Edition, 2010.
5. H. A. Taha, Operations Research: An Introduction, 10th ed., Pearson Education Limited, 2017.
6. Kanti Swarup, P. K. Gupta and Man Mohan, Operations Research, Sultan Chand & Sons, New Delhi, 2014.
7. S. S. Rao, Optimization Theory and applications, Halsted Press, 1984.
8. M.J. Schniederjans, Goal Programming: Methodology and Applications, Kluwer Academic Publ., 1995.

9. Y. J. Lai, C.L. Hwang, Fuzzy Multiple Objective Decision Making: Methods and Applications, Springer-Verlog, 1994.
10. A. Kendel, Fuzzy Mathematical Techniques with Applications, Addison-Wisely, 1986.

**Course Code: MATH5110**

**Course title:** Computational Differential Equations

**Course Credits: 3** (total contact hr. 45)

**Course Objectives:** The main purpose of this course is to introduce the basic issues in the numerical solution of differential equations by discussing some examples like Galerkin's method for the numerical solution of differential equations in the context of two basic problems from population dynamics and stationary heat.

**Course Contents:**

**Unit - I**

Introduction to Laplace and Fourier transformation, Solution of initial and boundary value problems using Laplace and Fourier transform, Galerkin's method with global and piecewise polynomials, solving linear algebraic systems.

**Unit - II**

Initial value problems for systems, The existence of solutions and Duhamel's formula, Solutions of autonomous and non-autonomous problems, Galerkin finite element methods, Error analysis and adaptive error control.

**Unit - III**

Poisson equation, Finite element method for the Poisson equation, Energy norm error estimates, Adaptive error control, Heat equation, Solution of the heat equation, Finite element method for the heat equation.

**Unit - IV**

Wave equation, Transport in one dimension, Wave equation in the higher dimensions, Stationary convection diffusion problems, Streamline diffusion method, A framework for an error analysis, Time dependent convection-diffusion problems.

**Reference Books:**

1. M. K. Jain, S. R. K. Iyenger and R. K. Jain, Computational Methods for Partial Differential Equations, Wiley Eastern, 1994.
2. K. Eriksson, D. Estep, P. Hansbo and C. Johnson, Computational Differential Equations, 2009.
3. M. K. Jain, Numerical Solution of Differential Equations, $2^{nd}$ ed., Wiley Eastern, 2018.
4. S. S. Sastry, Introductory Methods of Numerical Analysis, Prentice-Hall of India, 2002.
5. D. V. Griffiths and I. M. Smith, Numerical Methods for Engineers, Oxford University Press, 1993.
6. C. F. General and P. O. Wheatley, Applied Numerical Analysis, Addison- Wesley, 1998.
7. K. J. Bathe and E. L. Wilson, Numerical Methods in Finite Element Analysis, Prentice-Hall of India, 1987.

**Course Code: MATH5111**

**Course title:** Special Functions

**Course Credits: 3** (total contact hr. 45)

**Course Objectives:** The aim of this paper is to determine types of differential equations which may be solved by the application of special functions and analyse properties of special functions by their integral representations and symmetries.

**Course Contents:**

**Unit - I**

Legendre functions, Legendre polynomials, Recurrence relations for the Legendre polynomials, Series of Legendre polynomials, Legendre's differential equation, Neumann's formula for the Legendre functions, Recurrence relations for the functions, Use of associated Legendre functions in wave mechanics.

**Unit - II**

Bessel Functions, The origin of Bessel functions, Recurrence relations for the Bessel coefficients, Series expansions for the Bessel coefficients, Integral expressions for the Bessel coefficients, Bessel's differential equation, Spherical Bessel functions, Integrals involving Bessel functions, The modified Bessel functions, Asymptotic expansion of Bessel functions.

**Unit - III**

The Functions of Hermite and Laguerre, The Hermite polynomials, Hermite's differential equation, Hermite functions, The occurrence of Hermite functions in wave mechanics, The Laguerre polynomials, Laguerre's differential equation, The associated Laguerre polynomials and functions.

**Unit - IV**

Hypergeometric Functions, The hypergeometric series, An integral formula for the hypergeometric series, The hypergeometric equation, Linear relations between the solutions of the hypergeometric equation, Relations of contiguity, The confluent hypergeometric function, Generalised hypergeometric series.

**Reference Books:**

1. I. N. Sneddon, Special Functions of Mathematical Physics and Chemistry, 3rd ed., Longman Higher Education, 1980.
2. G. Andrews, R. Askey & R. Roy, Special Functions, Cambridge University Press, 2001.
3. L. Andrews, Special Functions for Engineers and Applied Scientists, $2^{nd}$ ed., Oxford University Press, 1998.
4. N. N. Lebedev, Special Functions & Their Applications, Dover Publications Inc., 2003.
5. W. W. Bell, Special Functions for Scientists and Engineers, Dover Publications Inc., 2004.

**Course Code: MATH5112**

**Course Title:** Theory of Elasticity

**Course Credits: 3** (total contact hr. 45)

**Course Objectives:** The main objective of this course is to study the branch of solid mechanics that deals with the effects of temperature change on solids' elastostatics and elastodynamics.

**Course Contents:**

**Unit - I**

Fundamentals of linear elasticity, Kinematics, Motion and equilibrium, Constitutive relations for isotropic and anisotropic body.

**Unit - II**

Formulation of problems of elasticity, Initial value problems and boundary value problems of elastostatics, Concept of an elastic state.

**Unit - III**

Variational formulation and solutions of elastostatics, Principle of minimum potential energy, Rayleigh-Ritz method, Vartiational principles.

**Unit - IV**

Variational formulation and solutions of elastodynamics, Hamilton-Kirchhoff principle, Gurtin's convolutional variational principle.

**Reference Books:**

1. Richard B. Hetnarski, Jozef Ignaczak: The Mathematical theory of elasticity, $2^{nd}$ ed, CRC press, Taylor & Francis Group, 2012.
2. Richard B. Hetnarski, M. Reza Eslami: Thermal stresses- Advanced theory and applications, $2^{nd}$ ed, Springer, 2019.
3. Martin H. Sadd, Elasticity: Theory, Application and Numerics, $2^{nd}$ed., Academic Press Inc, 2009.
4. L. D. Landau and E. M. Lifshitz, Theory of Elasticity (Course of Theoretical Physics), $2^{nd}$ed., Pergamon, 1981.
5. A. E. H. Love, A Treatise on the Mathematical Theory of Elasticity, Dover, 1944.
6. I. S. Sokolnikoff, Mathematical Theory of Elasticity, McGraw-Hill, 1946.
7. Y. B. Fu and R. W. Ogden, Nonlinear Elasticity: Application and Numerics, Cambridge University Press, 2001.

**Course Code: MATH5113**

**Course title:** Abstract Algebra

**Course Credits: 3** (total contact hr. 45)

**Course Objectives:** Fields forms one of the important and fundamental algebraic structures and has an extensive theory dealing mainly with field extensions which arise in the study of roots of polynomials. In this course we study Galois theory which is one of the most spectacular mathematical theories. It gives a beautiful connection of the theory of polynomial equations and group theory.

**Course Contents:**

**Unit - I**

Fields and their extension, Splitting fields, Fundamental theorem of field theory, Algebraic closure of a fields, Ruler and compass constructions.

**Unit - II**

Algebraic extension, Normal extension, Separability, Automorphism of field extension, Cyclotomic polynomials and extensions.

**Unit - III**

The fundamental theorem of Galois theory, Roots of unity, Finite fields, Primitive elements, Solvability by radicals.

**Unit - IV**

Normal series, Composition series of a group, Jordan-Holder theorem, Solvable group, Nilpotent groups.

**Reference Books**:

1. I.S. Luthar, I.B.S. Passi, Algebra Volume 4: Field Theory, Narosa Publishing House, 2008.
2. P.M. Cohn, Classic Algebra, John Wiley & Sons Ltd., 2000.
3. P.M. Cohn, Basic Algebra: Groups, Rings and Fields, Springer, 2005.
4. N. Jacobson, Basic Algebra, Volumes I & II, 2nd ed., Dover Publications, 2009.
5. T.W. Hungerford, Algebra, Springer-Verlag, 1981
6. D.S. Dummit and R.M. Foote, Abstract Algebra, 3rd ed., Wiley India Pvt. Ltd.,2011.

**Course Code: MATH5114**

**Course title:** Module Theory

**Course Credits: 3** (total contact hr. 45)

**Objective:** A module over a ring is a generalization of vector space over a field. The study of modules over a ring R provides us with an insight into the structure of R**.** Our aim is to realise the importance of rings and modules as central objects in algebra and to study some applications.

**Course Contents:**

**Unit - I**

Basic concepts of module theory, Quotient modules and Module homomorphisms, Generation of modules, Direct sum of modules, Free modules.

**Unit - II**

Exact sequences, Split exact sequences, Projective, injective and flat modules, Dual basis, Baer’s criterion, Divisible modules.

**Unit - III**

Tensor product of modules, Chain conditions, Hilbert basis theorem.

**Unit - IV**

Modules over principal ideal domains, Rational canonical form, Jordan canonical form, Semi-simple modules.

**Reference Books:**

1. M. F. Atiyah and I. G. MacDonald, Introduction to Commutative Algebra, CRC Press, Taylor & Francis, 2018.
2. P. M. Cohn, Classic Algebra, John Wiley & Sons Ltd., 2000.
3. P. M. Cohn, Basic Algebra: Groups, Rings and Fields, Springer, 2005.
4. D. S. Dummit and R. M. Foote, Abstract Algebra, 3rd ed., Wiley India Pvt. Ltd., 2011.

5. T. W. Hungerford, Algebra, Springer-Verlag, 1981.
6. N. Jacobson, Basic Algebra, Volumes I & II, 2nd ed., Dover Publications Inc., 2009.

**Course Code: MATH5115**

**Course Title:** Mathematical Modelling and its Applications

**Course Credits: 3** (total contact hr. 45)

**Course Objectives:** The overall goal of this course is to enable students to build mathematical models of real-world systems, analyze them and make predictions about behaviour of these systems. The construction and analysis of mathematical models would be inspired by real life problems. Variety of modelling techniques will be discussed with examples taken from physics, biology, chemistry, economics and other fields. The focus of the course will be on seeking the connections between mathematics and physical systems, studying and applying various modelling techniques to creating mathematical description of these systems, and using this analysis to make predictions about the system's behaviour.

**Course Contents:**

**Unit - I**

Idealization and general principle of model building, Classifications of mathematical models, Characteristics and limitations of mathematical models, Different types of mathematical models, Relevant mathematical techniques: Non-dimesionalization, Steady states, Linearization, Stability analysis, Phase diagram.

**Unit - II**

Non-linear growth and decay models, Mathematical modelling in dynamics through ordinary differential equations of first order: The Single Species population models, Exponential, Logistic, and Gromptz growth population, Constant Harvesting and bifurcations, Models with interacting populations: Lotka-Volterra Competition, Predator-prey interactions, Compartment models, Mathematical models through difference equations, Some simple models.

**Unit - III**

Mathematical modelling through difference equations in economics and finance, Mathematical modelling through difference equations in population dynamic and genetics, Situations that can be modelled through graphs, Mathematical models in terms of directed graphs.

**Unit - IV**

Mathematical models in terms of signed graphs, Mathematical models in terms of weighted digraphs, Mathematical modeling through linear programming, Linear programming models in forest management, Transportation and assignment models.

**Reference Books :**

1. J. N. Kapur, Mathematical Modelling, 2nd ed., New Age Intern, Pub., 2021.
2. Walter J. Meyer, Concept of Mathematical Modelling, McGraw-Hill, 1985.
3. F. Chorlton, Ordinary Differential and Difference Equations, Van Nostrand, 1965
4. J. D. Murray, Mathematical Biology-I: An Introduction, 3rd ed., Springer, 2008.

5. Lawrence Perko, Differential Equations and Dynamical Systems, Springer-Verlag, 2006.
6. Elizabeth S. Allman, John A. Rhodes, Mathematical Models in Biology: An introduction, Cambridge University Press, 2003.
7. Sandip Banerjee, Mathematical Modelling: Models, Analysis and Applications, CRC Press, 2014.

**Course Code: MATH5116**

**Course Title:** Coding Theory

**Course Credits: 3** (total contact hr. 45)

**Course Objectives:** The theory of error-correcting codes uses concepts from Algebra, Number Theory, Geometry and Probability Theory to ensure accurate transmission of information through noisy communication links. The objective of this course is to introduce basic concepts of coding theory: Encoding and decoding over finite fields, linear and cyclic codes, Hamming codes and many more.

**Course Contents:**

**Unit I**

Algebraic codes, Hamming weight, Generator and Parity-check matrices, Linear codes and their equivalence, Encoding and decoding of linear codes, Syndrome decoding, Constructions of new codes from given codes, Reed-Muller codes.

**Unit II**

Basic concepts of finite fields, Primitive elements, Cyclic codes, Cyclic codes as ideals, Generator polynomial of cyclic codes, Matrix representation of cyclic codes, Representation of cyclic codes by roots of unity.

**Unit III**

Burst error-correcting codes, Some special cyclic codes, BCH codes, Reed-Solomon Codes, Decoding algorithm for Reed-Solomon codes.

**Unit IV**

Bounds on codes: Hamming bound, BCH bound, Perfect codes, Hamming codes and Golay codes, Gilbert bound, Gilbert-Varshamov bound for linear codes, Singleton bound and MDS codes.

**Reference Books:**

1. S. Ling and C. Xing, Coding Theory: A First Course, Cambridge University Press, 2004.
2. R. Hill, A First Course in Coding Theory, Oxford University Press, 1986.
3. C. Huffman and V. Pless, Fundamentals of Error Correcting Codes, Cambridge University Press, 2010.
4. R. Lidl and H. Niederreiter, Finite Fields, 2nd ed., Cambridge University Press, 1997.

**Course Code: MATH5117**

**Course Title:** Differentiable Manifolds

**Credits Equivalent: 3** (total contact hr. 45)

**Course Objectives:** To understand the basics of differentiable manifolds, to make them understand the topics covered in allied courses like Algebraic Topology, Differential Topology, Riemannian geometry and Riemann-Finsler geometry.

**Course Contents:**

**Unit I:** Calculus in $\mathrm{R}^n$: tangent space, Vector fields, cotangent space and differentials on $\mathrm{R}^n$. Topological manifolds, Charts and atlases, differentiable manifolds, induced topology on manifolds, functions and maps, some special functions of class $\mathrm{C}^\infty$.

**Unit II:** Para compact manifolds and partition of unity, pullback functions, local coordinates systems and partial derivatives, differential of a map, the tangent bundle.

**Unit III:** Lie bracket, the cotangent space, the cotangent bundle, the dual of the differential map, one parameter group and vector fields.

**Unit IV:** Lie derivatives, tensors, tensor fields, connections, parallel translation, covariant differentiation of tensor fields, torsion tensor, curvature tensor, Bianchi and Ricci identities, geodesics, Riemannian manifolds.

**Reference Books:**


1. F. Warner, Foundations of Differentiable Manifolds and Lie Groups, Springer, New York, 1983.
2. J. M. Lee, Introduction to Smooth Manifolds, GTM, Vol. 218, Springer, New York, 2003.
3. L. Conlon, Differentiable Manifolds, 2nd ed., Birkhauser Boston, Cambridge, 2001.
4. L. W. Tu, An Introduction to Manifolds, 2nd ed., Springer, 2011.
5. N. J. Hicks, Notes of Differential Geometry, D. Van Nostrand Reinhold Company, New York, 1965.
6. S. Kumaresan, A Course in Differential Geometry and Lie Groups (Texts and Readings in Mathematics), Hindustan Book Agency, 2002.
7. S. S. Chern, W. H. Chen and K. S. Lam, Lectures on Differential Geometry, World Scientific Publishing Co. Pvt. Ltd., 2000.
8. W. M. Boothby, An Introduction to Differentiable Manifolds and Riemannian Geometry, 2nd edition, Academic Press, New York, 2003.
9. R. Abraham, J. E. Marsden, T. Ratiu, Manifolds, Tensor Analysis and Applications, 2nd ed., Springer, 1988.
10. B. Spain, Tensor Calculus: A Concise Course, Dover Publications, 2003.


*******The End*******